Name __Solutions____ Discrete I, Quiz 9

Show using induction that $1 \cdot 2 \cdot 3 \cdot \cdots \cdot n \cdot (n+1) \geq 2^n$ for all $n = 0,1,2,\ldots$

Base case: $1 \overset{?}{\geq} 2^0$ That is, $1 \geq 1$.

Induction Hypothesis: Assume that:

$$1 \cdot 2 \cdot \cdots \cdot k \cdot (k+1) \geq 2^k.$$

Induction step: (Here we show the case $n = k+1$)

$$1 \cdot 2 \cdot 3 \cdot \cdots \cdot k \cdot (k+1) \cdot (k+2) \geq 2^k \cdot (k+2) \geq 2^k \cdot 2 = 2^{k+1}$$

Indeed, we have proven that $1 \cdot 2 \cdot 3 \cdot \cdots \cdot n \cdot (n+1) \geq 2^n$ for all $n = 0, 1, 2, \ldots$