

Show using induction that $6|7^n - 1$ for all $n = 2, 3, 4, \dots$

Base case: $7^2 - 1 = 48 = 6 \cdot 8$, so indeed $6|7^2 - 1$.

Induction hypothesis: Assume $6|7^k - 1$ for some k .

Induction step: From the induction hypothesis, we know that there is some m such that $7^k - 1 = 6m$.
Hence we have:

$$\begin{aligned}7^{k+1} - 1 &= 7 \cdot 7^k - 1 \\ &= 7^k - 1 + 6 \cdot 7^k \\ &= 6m + 6 \cdot 7^k \\ &= 6(m + 7^k)\end{aligned}$$

Thus $6|7^{k+1} - 1$.

Therefore $6|7^n - 1$ for all $n = 2, 3, 4, \dots$