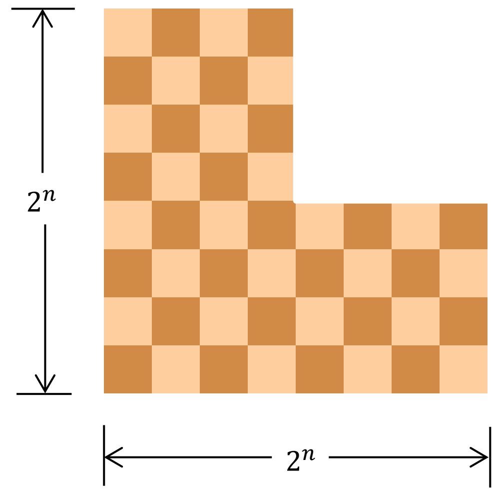
Name ___Solutions____ Discrete I, Quiz 11

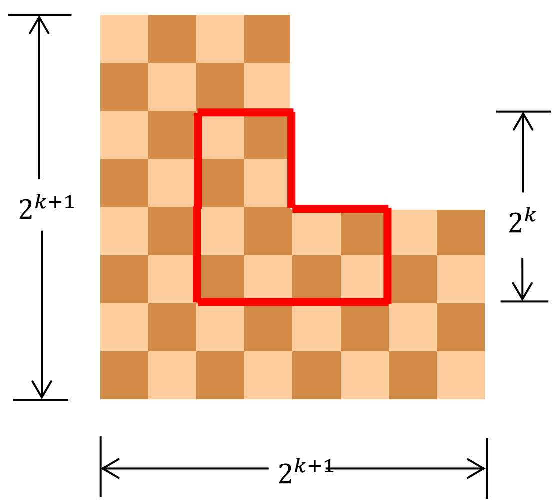
Show using induction that any "L-shaped" board as shown below can be tiled by the tromino shown below. The size of the board can be any $2^n \times 2^n$ for $n = 1, 2, 3, \ldots$



The base case is trivial: a $2^1 \times 2^1$ grid is just a single tile:

Induction Hypothesis: Assume that any $2^k \times 2^k$ board can be tiled using the tile.

Induction step: Suppose we have a $2^{k+1} \times 2^{k+1}$ board. We will sub-divide it as shown below.



The image here isn't quite right, as it appears to be 8×8. By hand I would draw it with parts left out, illustrating that k is arbitrary and the size could be anything.

As can be seen from the diagram, we now have four $2^k \times 2^k$ "L-shaped" boards. These can be tiled via the induction hypothesis, and together they give a tiling of the new board.

Therefore all "L-shaped" boards of size $2^n \times 2^n$ can be tiled, no matter what positive integer n is.