

For each of the following, determine whether the function f is one-to-one. For those that are not, provide a short justification. When not specified, the domain and codomain are \mathbb{R} .

1. $f(x) = 2x + 5$

This is one to one, as can be seen below.

Suppose $f(x_1) = f(x_2)$

$$\therefore 2x_1 + 5 = 2x_2 + 5$$

$$\therefore 2x_1 = 2x_2$$

$$\therefore x_1 = x_2$$

2. $f(x) = 2x^2$

This is not one to one, as can be seen below.

$$f(-1) = 2(-1)^2 = 2 = 2(1)^2 = f(1)$$

3. $f(x) = x^2$ with domain $[0, \infty)$

This is one to one, as can be seen below.

Suppose $f(x_1) = f(x_2)$

$$\therefore x_1^2 = x_2^2$$

$$\therefore x_1 = \pm x_2$$

$$\therefore x_1 = x_2 \quad (\text{Getting here used the fact that both } x_1 \text{ and } x_2 \text{ are nonnegative})$$

4. $f(x) = \lfloor x \rfloor$

This is not one to one, as can be seen below.

$$f(1.6) = \lfloor 1.6 \rfloor = 1 = \lfloor 1.2 \rfloor = f(1.2)$$

5. $f(x) = \lfloor x \rfloor$ with domain $\{0, 1, 2, \dots\}$

This is not one to one, as for integers x and y , $\lfloor x \rfloor = \lfloor y \rfloor$ iff $x = y$.