Name __Solutions_____ Discrete I, Quiz 19

Consider the two relations given below on $\mathbb{R}$.

$$xRy \text{ iff } x \leq y$$
$$xSy \text{ iff } y - x \geq 2$$

Choose and complete 2 of the following problems. Justify all answers.

1) Is R reflexive?
2) Is R symmetric?
3) Is R antisymmetric?
4) Is R transitive?
5) Is R total?

R is reflexive as $x \leq x$ for all real x.

R is not symmetric. Consider $x = 1$ and $y = 2$. Then $1 \leq 2$ but not $2 \leq 1$.

R is antisymmetric. If $x \leq y$ and $y \leq x$, then $x = y$.

R is transitive. If $x \leq y$ and $y \leq z$, then $x \leq z$.

R is total: consider any two real numbers x and y. If $x \leq y$ we're done. If not then $x > y$, so $y \leq x$.

Choose and complete 2 of the following problems. Justify all answers.

1) Is S reflexive?
2) Is S symmetric?
3) Is S antisymmetric?
4) Is S transitive?
5) Is S total?

S is not reflexive. Consider $x = 1$, then $x - x = 0 \not\geq 2$

S is not symmetric. Consider $x = 1$ and $y = 4$. Then $4 - 1 \geq 2$ so $1S4$. However, $1 - 4 \not\geq 2$, so not $4S1$.

S is antisymmetric. Consider the equations $y - x \geq 2$ and $x - y \geq 2$. These can never both be true at the same time, in particular look at the first equation multiplied by negative 1: $x - y < -2$.

S is transitive. Suppose xSy and ySz. That is to say that $y - x \geq 2$ and $z - y \geq 2$. Adding these we get:

$$y - x + z - y \geq 4$$
$$\therefore z - x \geq 4 \geq 2$$
$$\therefore xSz$$

S is not total. Consider for instance 1 and 2. Neither $1 - 2 \geq 2$ nor $2 - 1 \geq 2$.