Name $\qquad$ Solutions $\qquad$

Use the code below to answer the following questions.
$x=17$
for $i$ from 0 to $n-1$
$x=x *(x-2)$

1) Assuming all arithmetic can be done in hardware, what is the asymptotic runtime of this algorithm?

$$
O(n)
$$

2) Assuming all arithmetic can be done in hardware, what is the asymptotic space requirement of this algorithm?

$$
O(1)
$$

3) If $n$ is large enough that the arithmetic needs to be done in software, what is the asymptotic space requirement of this algorithm?

If $m$ is the size of the largest value that $x$ gets to, it is

$$
O(\log (m))
$$

Exactly what this is, is unclear. To find an upper bound, let us assume that the third line is " $x=x^{*} x^{\prime}$ ". In this case we construct the table below to see the first four values of $n$ :

| $n$ | $x$ |
| :---: | :---: |
| 1 | $17^{2}$ |
| 2 | $17^{4}$ |
| 3 | $17^{8}$ |
| 4 | $17^{16}$ |

From this we see that the value of $x$ is $O\left(17^{2^{n}}\right)$. Hence the space requirement is:

$$
O\left(\log \left(17^{2^{n}}\right)\right)=O\left(2^{n} \cdot \log (17)\right)=O\left(2^{n}\right)
$$

