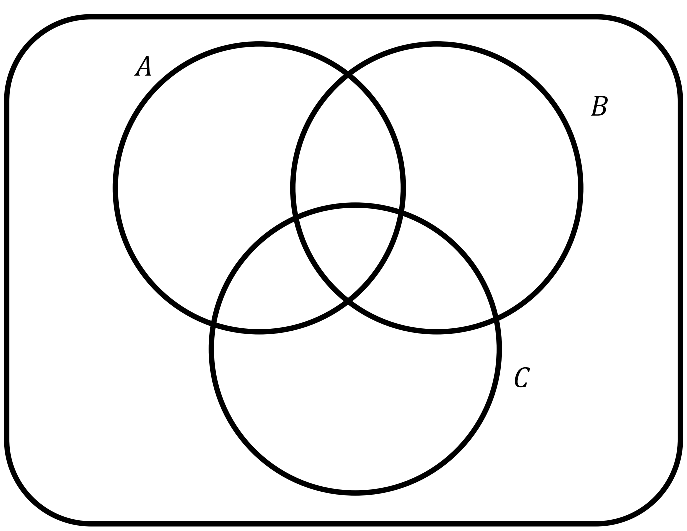
Name ____ Discrete I, Test 1, 9/19/2014

1) Given that p is false, q is true, and r is false, what is $p \wedge (q \vee \neg r) \wedge (p \vee \neg r)$? (5 points)

2) Find the truth table for $(p \vee q) \Rightarrow (p \wedge q)$. (5 points)

T or F 3) $\{x\} \subseteq \{x\}$
T or F 4) $\{x\} \in \{x\}$
T or F 5) $\{x\} \in \{x, \{x\}\}$
T or F 6) $\{x\} \subseteq \{x, \{x\}\}$
T or F 7) $\{x\} \subseteq \mathcal{P}(\{x, y\})$
(1 point each)

8) In the Venn Diagram below, shade the region corresponding to $A \cap (C \cup B)^c$. (5 points)



9) Prove, using induction, the following equality: (20 points)

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

T or F 10) $\forall_{x\in\mathbb{R}}(x^2 \geq 0)$
T or F 11) $\exists_{x\in\mathbb{R}}(x^2 \geq 0)$
T or F 12) $\forall_{x\in\mathbb{R}}(x^2 = 0)$
T or F 13) $\exists_{x\in\mathbb{R}}(x^2 = 0)$
T or F 14) $\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}\exists_{z\in\mathbb{R}}(xyz + z^2 = 1)$
(1 point each)

15) Simplify $\neg\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}\exists_{z\in\mathbb{R}}(xyz + z^2 = 1)$ (5 points)

16) Justify the claim that the sum of two odd numbers is even. (5 points)

17) Disprove the claim that the sum of three odd numbers is even. (5 points)

18) Briefly Explain the problem with the following supposed proof: (10 points)

I claim that $-2 = 2$.

First I square both sides: $(-2)^2 = (2)^2$

Simplifying this, we see that $4 = 4$, which is true.

Therefore, $-2 = 2$.

19) Write a complete sentence that is an example of an implication. (5 points)

20) How many rows would the truth table for $(p \vee q) \wedge (r \vee s) \wedge (x \vee y)$ have? (5 points)

21) Below are two supposed induction proofs. Both of them are flawed. Choose ONE of them and explain or identify what the error is. (20 points)

We prove the following statement:

$$\sum_{i=1}^{n} i = \frac{n^2 + n + 2}{2}$$

Assume the statement holds for some arbitrary k:

$$\sum_{i=1}^{k} i = \frac{k^2 + k + 2}{2}$$

Now we show that it is true for the next index, $k + 1$:

$$\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1) = \frac{k^2 + k + 2}{2} + \frac{2k + 2}{2} = \frac{k^2 + 3k + 4}{2} = \frac{(k+1)^2 + (k+1) + 2}{2}$$

Therefore, for all integers $n = 1, 2, 3, \dots$ we have

$$\sum_{i=1}^{n} i = \frac{n^2 + n + 2}{2}$$

OR

We prove that all horses are the same color. Clearly this is true if there is only one horse.

Assume that all sets of exactly k horses are the same color. Then consider any set of $k + 1$ horses. By assumption the first k horses must be the same color, say c_1. Also by assumption the last k horses must be the same color, say c_2. Since the set of the first k horses and the set of the last k horses overlap by the middle $k - 1$ horses, the two color groups must be the same: $c_1 = c_2$. Hence all $k + 1$ horses are the same color.

Therefore, we have proven that all horses are the same color.