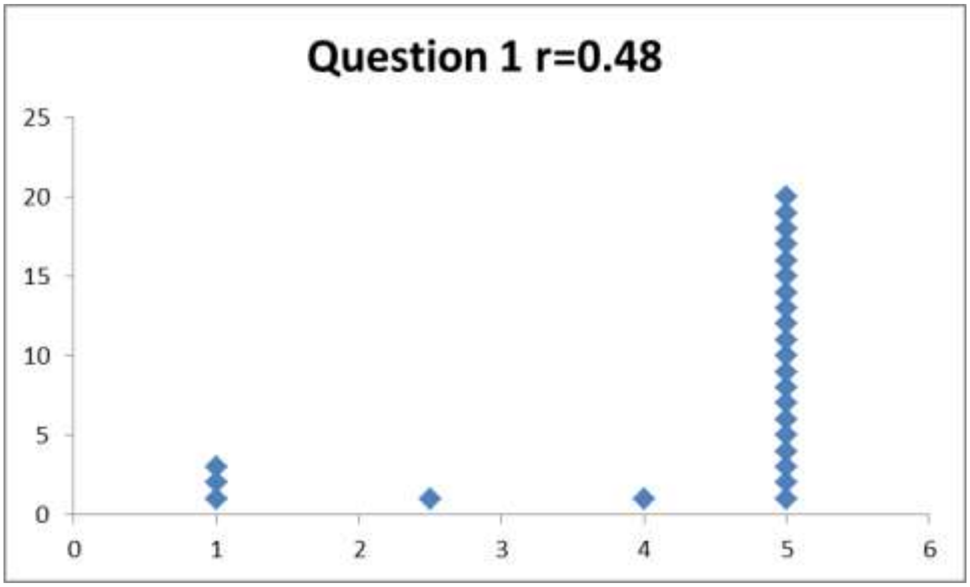
Name ___Solutions____ Discrete I, Test 1, 9/19/2014

1) Given that p is false, q is true, and r is false, what is $p \wedge (q \vee \neg r) \wedge (p \vee \neg r)$? (5 points)

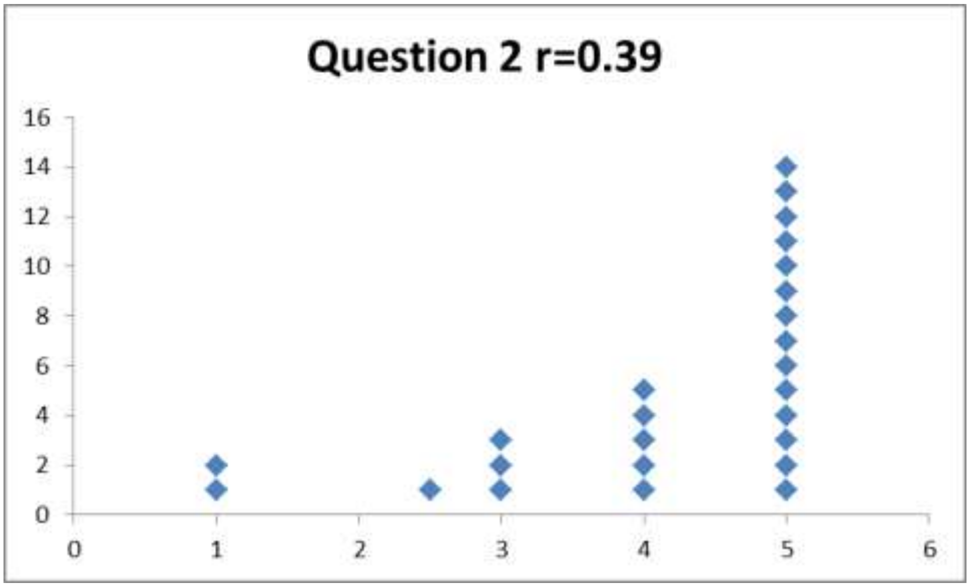
False

You can plug everything in, or notice that it reads "False and ______"

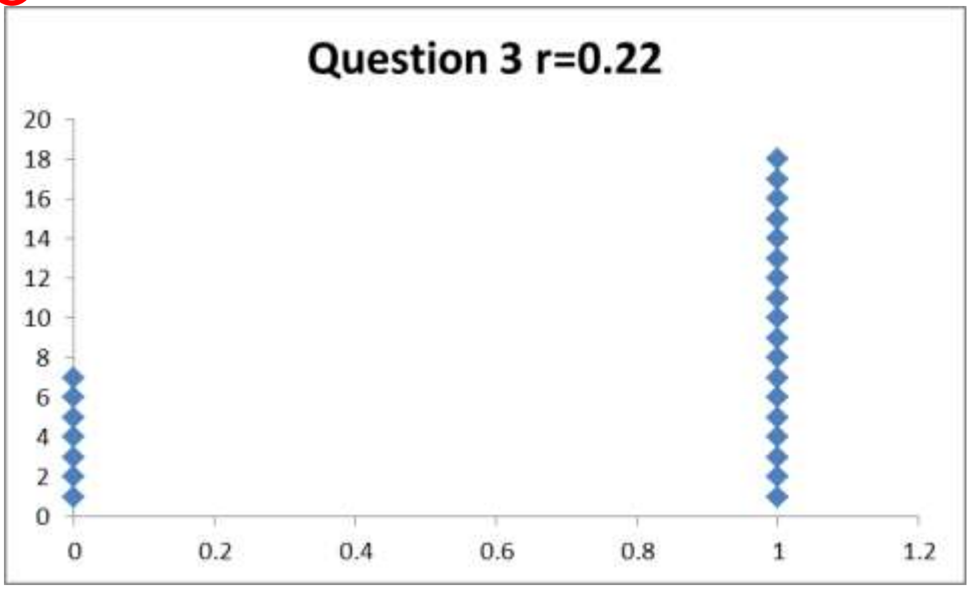


2) Find the truth table for $(p \vee q) \Rightarrow (p \wedge q)$. (5 points)

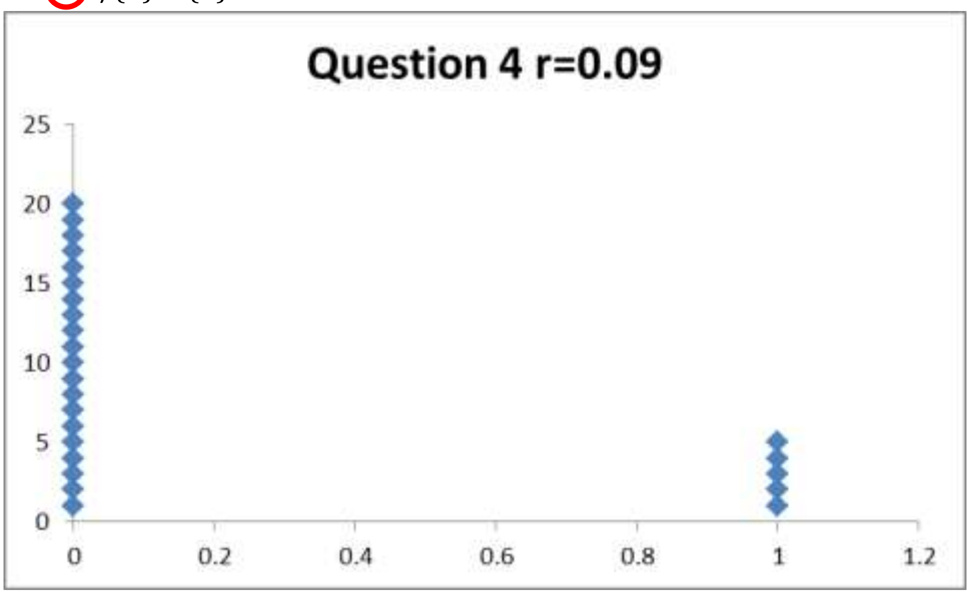
p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \Rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T



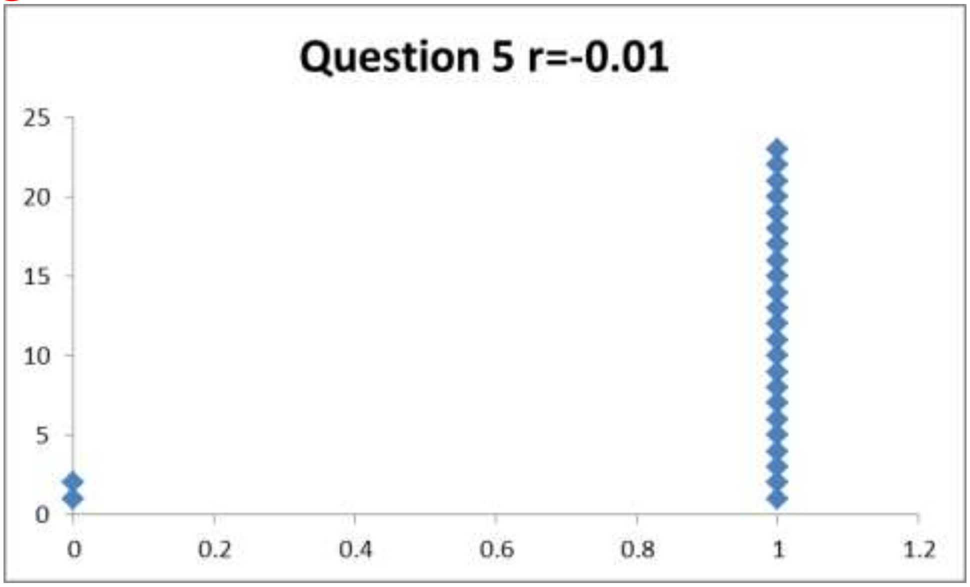
T or F 3) $\{x\} \subseteq \{x\}$



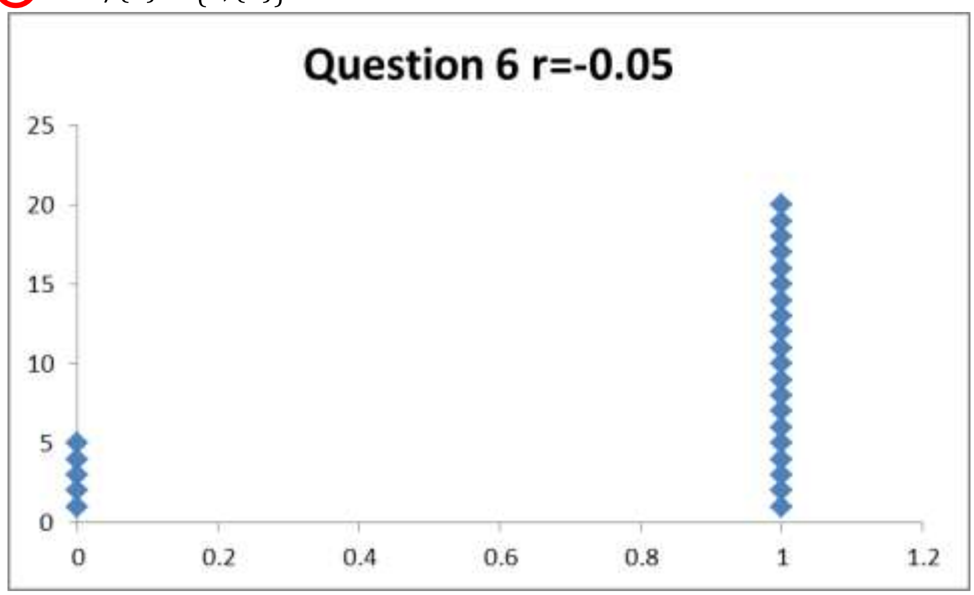
T or F 4) $\{x\} \in \{x\}$



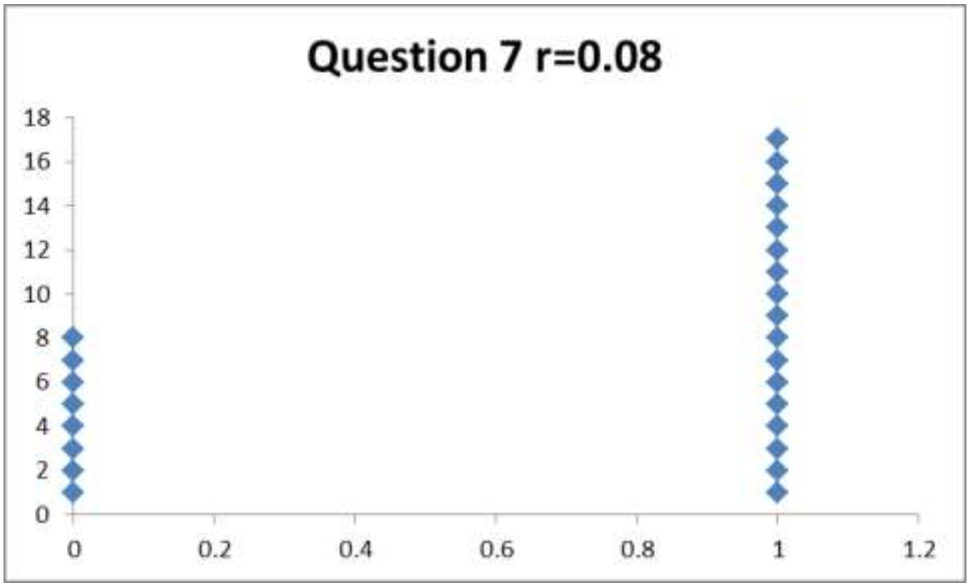
T or F 5) $\{x\} \in \{x, \{x\}\}$



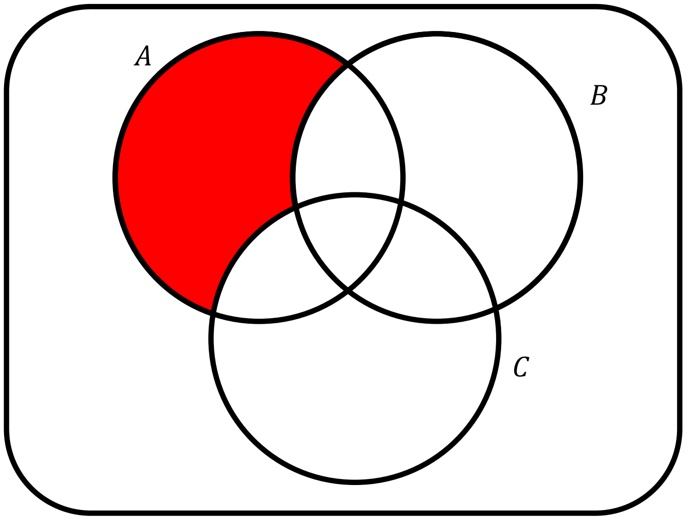
T or F 6) $\{x\} \subseteq \{x, \{x\}\}$



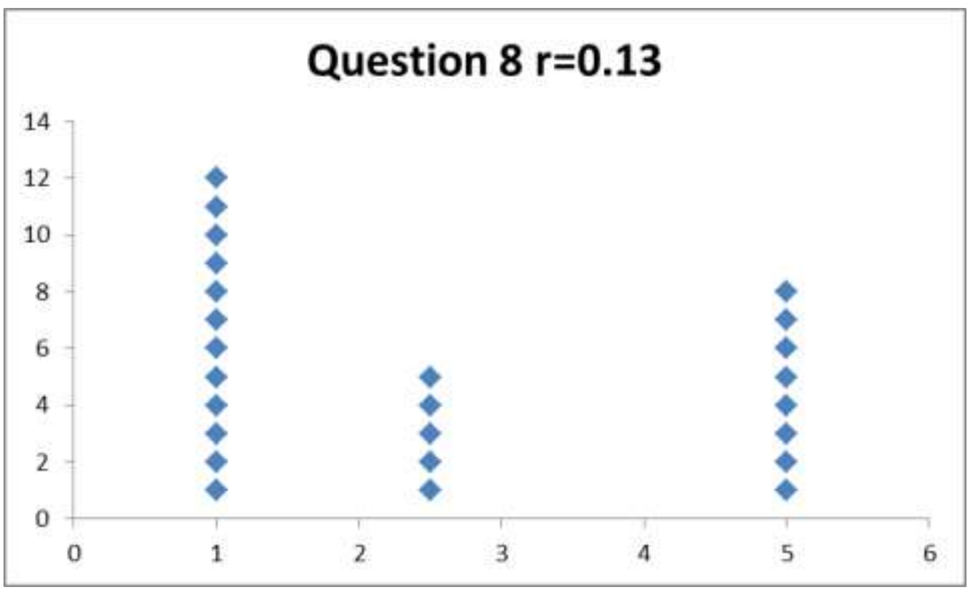
T or F 7) $\{x\} \subseteq \mathcal{P}(\{x, y\})$



8) In the Venn Diagram below, shade the region corresponding to $A \cap (C \cup B)^c$. (5 points)



Half credit was given as long as it was a subset of A.



9) Prove, using induction, the following equality for $n = 0, 1, 2, \ldots$ (20 points)

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Base case: If $n = 0$ then both the left and right hand sides are zero: $0^2 = \frac{0 \cdot 2}{6}$

Induction hypothesis: For some k, assume the equality $\sum_{i=0}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$

Inductive step: We now show that the equality is true in the $(k+1)^{th}$ case:

$$\sum_{i=0}^{k+1} i^2 = \sum_{i=0}^{k} i^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right] = (k+1)\left[\frac{k(2k+1)}{6} + \frac{6(k+1)}{6}\right]$$

$$= (k+1)\left[\frac{2k^2 + 7k + 6}{6}\right] = (k+1)\left[\frac{(k+2)(2k+3)}{6}\right]$$

$$= \frac{(k+1)\big((k+1)+1\big)\big(2(k+1)+1\big)}{6}$$

Hence by induction,

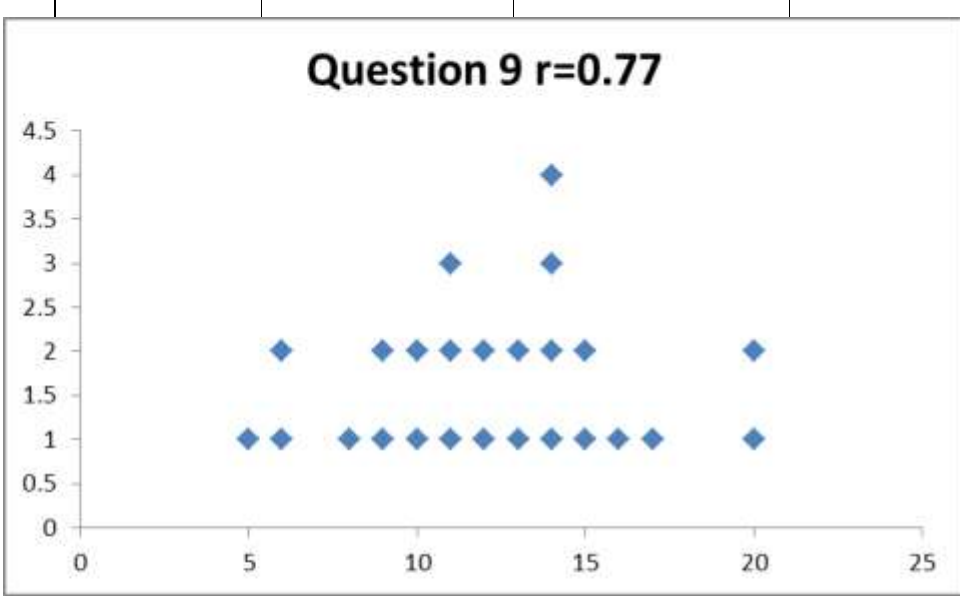
$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

for all $n = 0, 1, 2, \ldots$

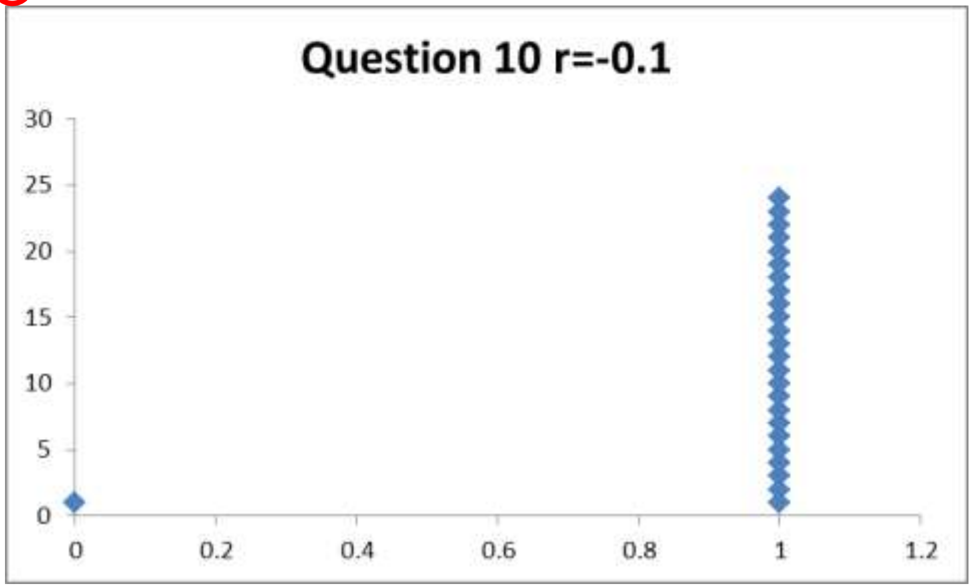
Base Case		
0 points	1 point	2 points
Did not check the base case	Checked the base case	Clearly checked the base case

Induction Hypothesis					
0 points	1 point	3 points	4 points	5 points	6 points
Missing	Some progress such as an assumption	Incorrect equation Correct assumption	Correct equation No assumption	Correct equation Incorrect assumption	Correct equation Correct assumption

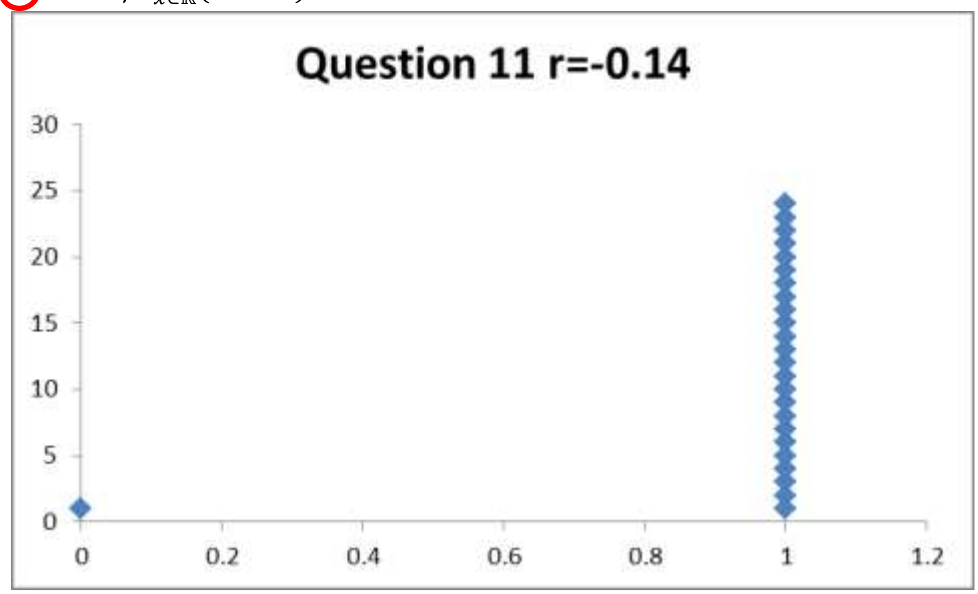
Inductive Step				
0 points	3 points	6 points	9 points	12 points
0 of the 4	1 of the 4	2 of the 4	3 of the 4	Correct logical flow Correctly applied IH Correct algebra Correct goal



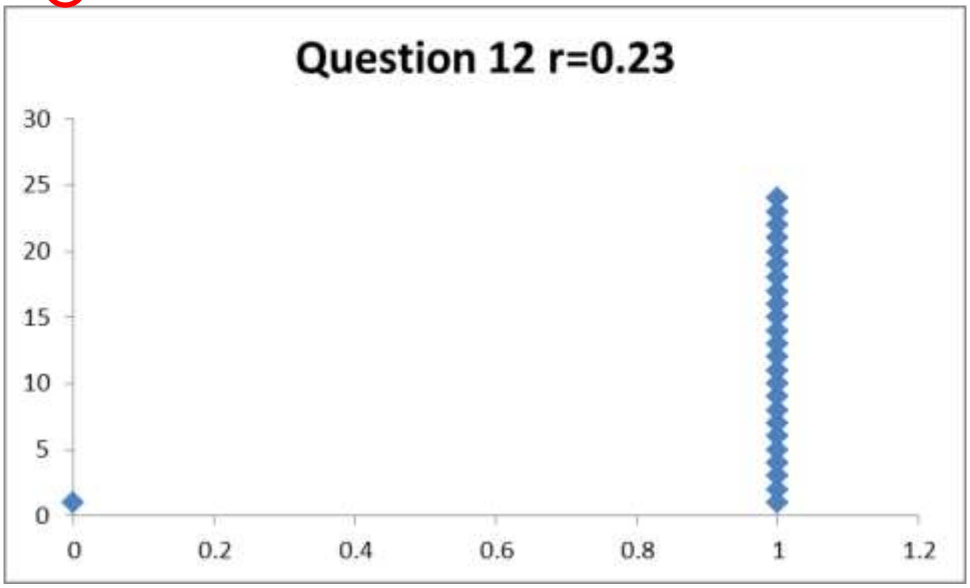
T or F 10) $\forall_{x \in \mathbb{R}} (x^2 \geq 0)$



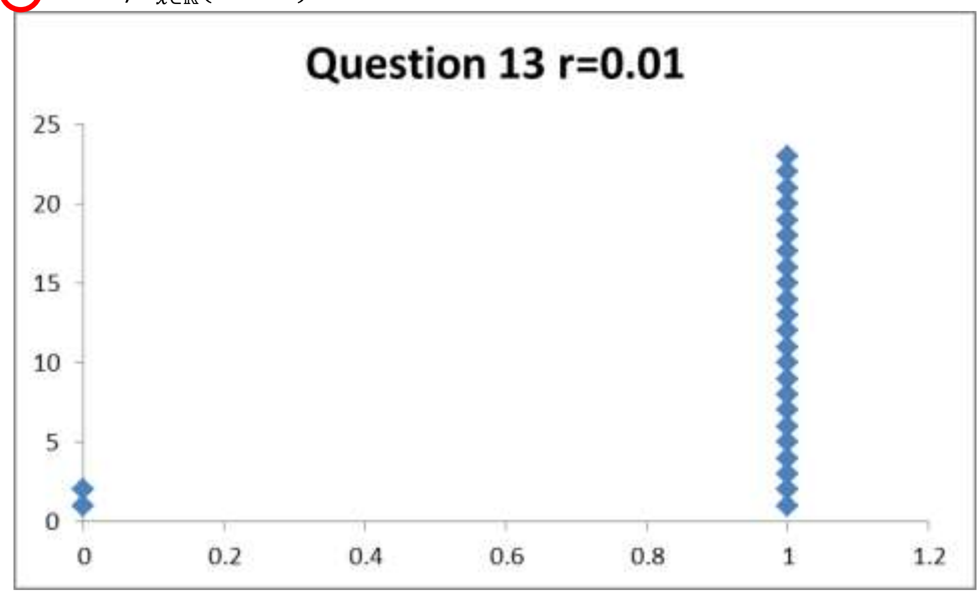
T or F 11) $\exists_{x \in \mathbb{R}} (x^2 \geq 0)$



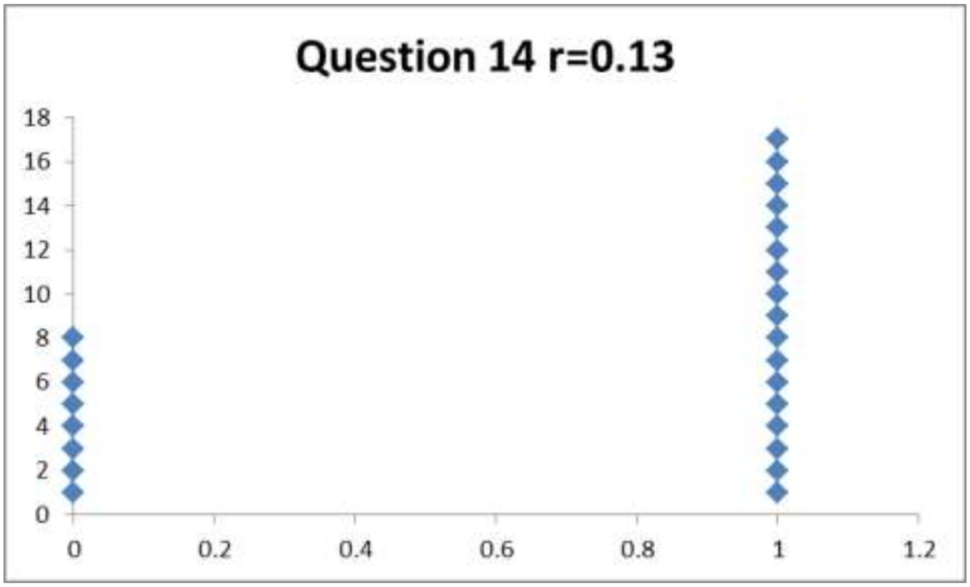
T or F 12) $\forall_{x \in \mathbb{R}}(x^2 = 0)$



T or F 13) $\exists_{x \in \mathbb{R}}(x^2 = 0)$

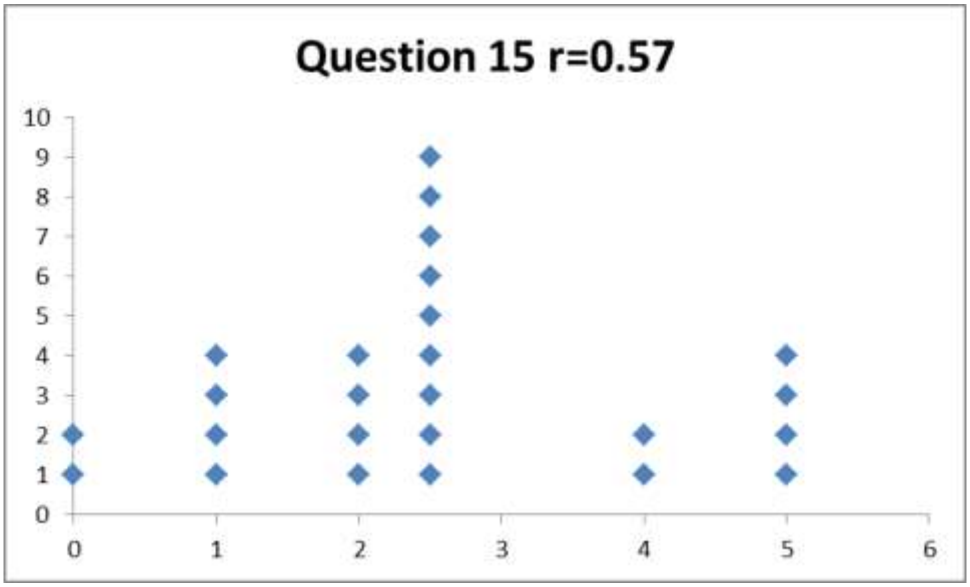


T or F 14) $\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}\exists_{z\in\mathbb{R}}(xyz+z^2=1)$



15) Simplify $\neg\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}\exists_{z\in\mathbb{R}}(xyz+z^2=1)$ (5 points)

$$\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}\forall_{z\in\mathbb{R}}(xyz+z^2\neq 1)$$



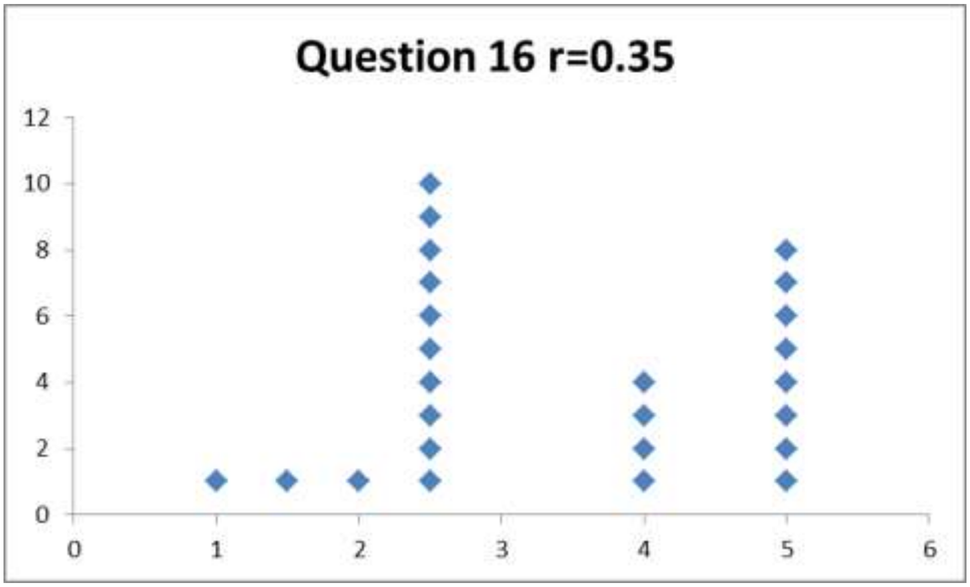
16) Justify the claim that the sum of two odd numbers is even. (5 points)

Two arbitrary odd numbers look like $2k + 1$ and $2l + 1$. Adding these we get:

$$2k + 1 + 2l + 1 = 2(k + l + 1)$$

…lots of people just did an example.

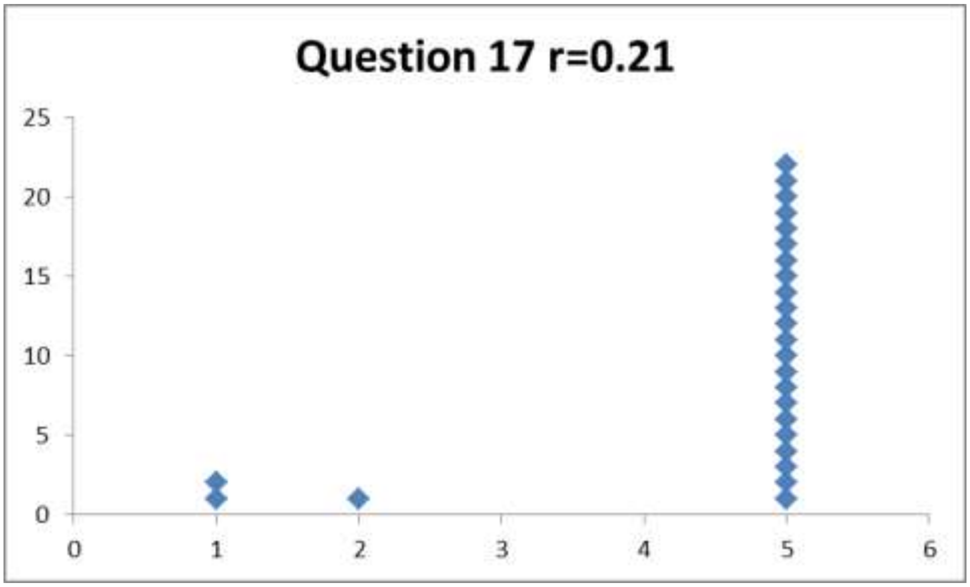
…Two people didn't even add two numbers.



17) Disprove the claim that the sum of three odd numbers is even. (5 points)

$$1 + 1 + 1 = 3$$

…Two people “proved” that the sum of three odd numbers is even.



18) Briefly Explain the problem with the following supposed proof: (10 points)

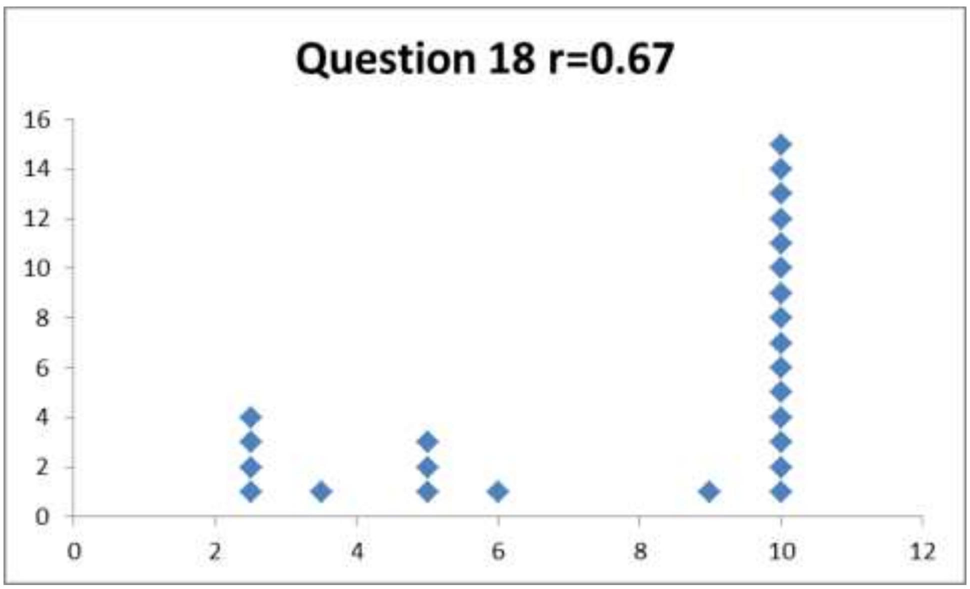
I claim that $-2 = 2$.

First I square both sides: $(-2)^2 = (2)^2$

Simplifying this, we see that $4 = 4$, which is true.

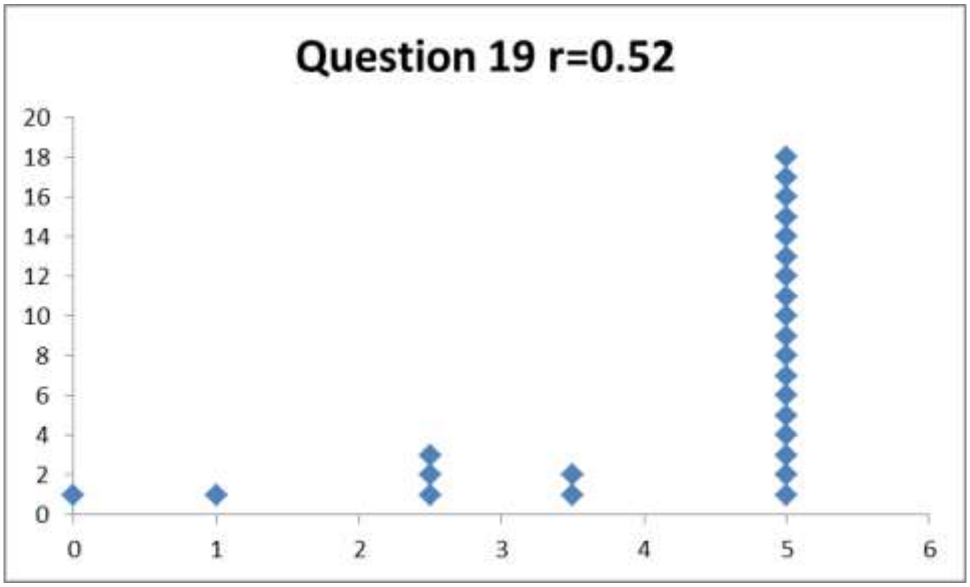
Therefore, $-2 = 2$.

This is circular. We started with what we're trying to prove!



19) Write a complete sentence that is an example of an implication. (5 points)

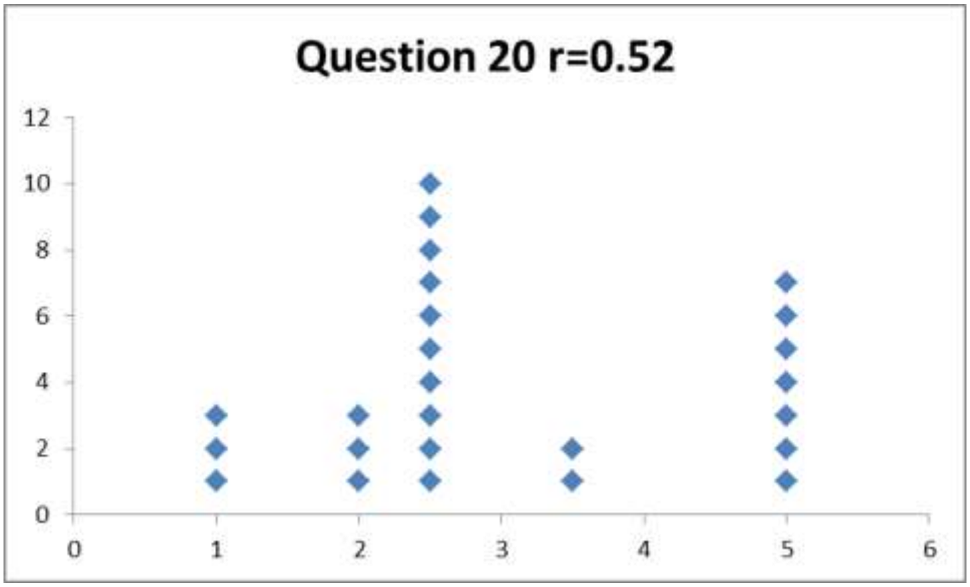
If this sentence is an implication, then it is correct.



20) How many rows would the truth table for $(p \vee q) \wedge (r \vee s) \wedge (x \vee y)$ have? (5 points)

$$2^6 = 64$$

The answer should at least be a power of 2, because every variable doubles the size of the table.



21) Below are two supposed induction proofs. Both of them are flawed. Choose ONE of them and explain or identify what the error is. (20 points)

We prove the following statement:

$$\sum_{i=1}^{n} i = \frac{n^2 + n + 2}{2}$$

Assume the statement holds for some arbitrary k:

$$\sum_{i=1}^{k} i = \frac{k^2 + k + 2}{2}$$

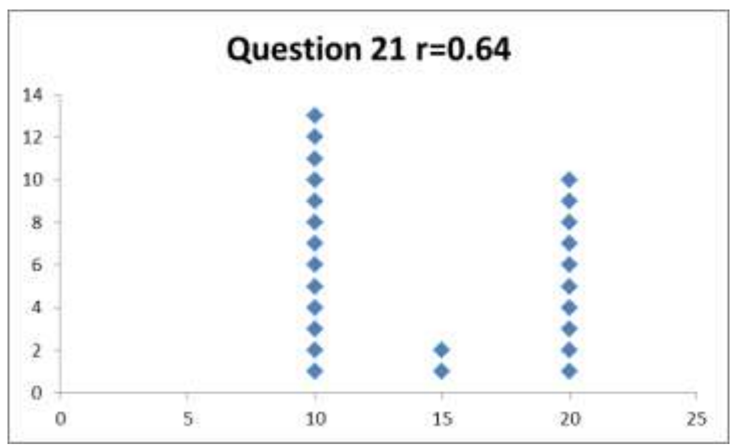
Now we show that it is true for the next index, $k + 1$:

$$\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1) = \frac{k^2 + k + 2}{2} + \frac{2k + 2}{2} = \frac{k^2 + 3k + 4}{2} = \frac{(k+1)^2 + (k+1) + 2}{2}$$

Therefore, for all integers $n = 1, 2, 3, \ldots$ we have

$$\sum_{i=1}^{n} i = \frac{n^2 + n + 2}{2}$$

OR



We prove that all horses are the same color. Clearly this is true if there is only one horse.

Assume that all sets of exactly k horses are the same color. Then consider any set of $k + 1$ horses. By assumption the first k horses must be the same color, say c_1. Also by assumption the last k horses must be the same color, say c_2. Since the set of the first k horses and the set of the last k horses overlap by the middle $k - 1$ horses, the two color groups must be the same: $c_1 = c_2$. Hence all $k + 1$ horses are the same color.

Therefore, we have proven that all horses are the same color.

The first supposed proof doesn't have a base case. (But the IH works fine)

The second proof's inductive step doesn't work for $k = 2$ because the two sets don't overlap.