1) Show that $3n + 1 < 3^n$ for all $n = 6, 7, 8, \ldots$ (20 points)
2) Show that a $2^n \times 2^n$ chess board with one random tile removed can be tiled with the piece shown below. (20 points)
3) Show that all integers \( n \geq 15 \) can be written as:

\[ n = 3x + 7y \]

where \( x \) and \( y \) are integers. (20 points)
4) Let $f(x) = [3x]$ with domain $\mathbb{R}$. What is the range of $f$? (5 points)

5) Is the function $f(x) = 3x + 1$ one to one? Justify your answer. (10 points)

6) Decompose the function $f(x) = (2x + 4)^3$ into two simpler functions. Be sure to explicitly give the rule for both of your answers. (5 points)
7) Reindex the summation below to start at \( i = 0 \). (5 points)
\[
\sum_{i=3}^{n+7} 2^i
\]

8) Give an example of a sequence that satisfies each of the following: (5 points)
   - Nondecreasing
   - Not increasing
   - Not constant

9) List all strings of length at most 2 that are members of \( \{a, b\}^* \). (5 points)

10) List all the substrings of the string “abc”. (5 points)
11) Let $H_n$ be the $n^{th}$ harmonic number as defined below.

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

Show the following for all $m = 1, 2, 3, \ldots$

$$\sum_{n=1}^{m} H_n = (n + 1)H_n - n$$

(20 bonus points)