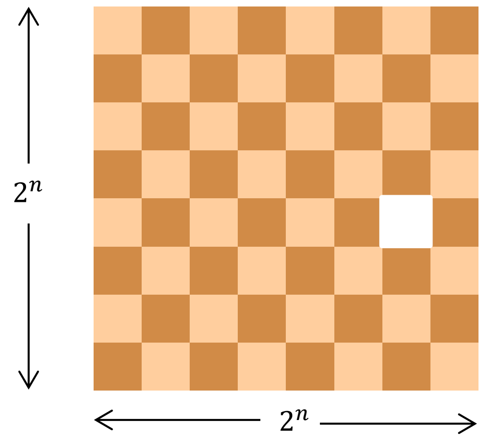
Name ____ Discrete I, Test 1, 10/24/2014

1) Show that $3n + 1 < 3^n$ for all $n = 6, 7, 8, \ldots$ (20 points)

2) Show that a $2^n \times 2^n$ chess board with one random tile removed can be tiled with the piece shown below. (20 points)



3) Show that all integers $n \geq 15$ can be written as:

$$n = 3x + 7y$$

where x and y are integers. (20 points)

4) Let $f(x) = \lceil 3x \rceil$ with domain $\mathbb{R}$. What is the range of f? (5 points)

5) Is the function $f(x) = 3x + 1$ one to one? Justify your answer. (10 points)

6) Decompose the function $f(x) = (2x + 4)^3$ into two simpler functions. Be sure to explicitly give the rule for both of your answers. (5 points)

7) Reindex the summation below to start at $i = 0$. (5 points)

$$\sum_{i=3}^{n+17} 2^i$$

8) Give an example of a sequence that satisfies each of the following: (5 points)

Nondecreasing

Not increasing

Not constant

9) List all strings of length at most 2 that are members of $\{a, b\}^*$. (5 points)

10) List all the substrings of the string "`abc`". (5 points)

11) Let H_n be the n^{th} harmonic number as defined below.

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

Show the following for all $m = 1, 2, 3, \ldots$

$$\sum_{n=1}^{m} H_n = (n+1)H_n - n$$

(20 bonus points)