Name _____ Discrete I, Test 3, 12/1/2014

1) Draw the digraph of the relation $\{(a,a),(a,b),(a,c),(b,a)\}$ (5 points)

For the rest of the problems on this page, use the relation R on $\mathbb{R}$ defined by $(x,y) \in R$ if and only if $x - y = 2$

2) Is R reflexive? Justify your answer. (5 points)

3) Is R symmetric? Justify your answer. (5 points)

4) Is R transitive? Justify your answer. (5 points)

5) Find a partition of $\{a, b, c, d, e, f, g\}$ with 3 parts. (5 points)

Compute each of the following mod 13.

6) $10 + 8$ (5 points)

7) $7 - 11$ (5 points)

8) Solve $7x \equiv 4 \pmod{13}$. (5 points)

Use the following pseudocode to answer the questions on this page. Assume `n` is the length of the string `s`.

```
count = 0
    for i from 0 to n − 1
        if s_i == 'd'
            count = count + 1
            s_i = 'T'
return s, count
```

9) For the input `s="abcdedcba"` what is the return value for `count`? (5 points)

10) For the same input, what is the return value for `s`? (5 points)

11) For the same input, exactly how many comparisons **of any kind** are performed? (5 points)

12) For the same input, exactly how many times is `count` assigned a value? (5 points)

13) What is the "big-oh" growth rate of the function $f(n) = 2n^3 + 4n^2 - 5n$? (5 points)

14) Call your answer to the previous question $g(n)$. Justify your answer to the previous by finding the constant multiple and point that it starts to apply: (Fill in the boxes; show and supporting work or derivation below) (10 points)

$$f(n) \leq \boxed{} \cdot g(n) \text{ whenever } n \geq \boxed{}$$

15) Consider the code below. If "Line 3" is the line of interest and everything else is trivial, what is the asymptotic growth rate of this algorithm? (5 points)

```
for i from 0 to n-1
   for j from 0 to i
      "Line 3"
```

Use the code below to answer questions on this page.

```
x = 1
for i from 0 to n-1
   x = x * 5
```

16) Assuming all arithmetic can be done in hardware, what is the asymptotic runtime of this algorithm?
(5 points)

17) Assuming all arithmetic can be done in hardware, what is the asymptotic space requirement of this algorithm?
(5 points)

For the next two problems assume that we have a multiplication algorithm that requires $\Theta(m \log(m))$ runtime and $\Theta(\log(m))$ space to multiply two m-bit numbers.

18) If n is large enough that the arithmetic needs to be done in software, what is the asymptotic space requirement of this algorithm?
(10 points – 5 for the answer, 5 for the simplification/derivation)

19) If n is large enough that the arithmetic needs to be done in software, what is a bound on the asymptotic runtime of this algorithm?
(10 bonus points)