Name _Solutions____ Discrete I, Quiz 3

1) Let P be the set of passwords, and Q be the set of computers. Choose ONE the following statements and express it in statement in a grammatically correct sentence. (4 points)

$\exists_{x\in P}\forall_{y\in Q}(x \text{ unlocks } y)$ $\quad$ $\exists_{x\in P}\exists_{y\in Q}(x \text{ unlocks } y)$ $\quad$ $\forall_{y\in Q}\forall_{x\in P}(x \text{ unlocks } y)$

$\exists_{x\in P}\forall_{y\in Q}(x \text{ unlocks } y)$
"There is a password such that for all computers, the password unlocks the computer"
"There is a universal password that unlocks all computers"

$\exists_{x\in P}\exists_{y\in Q}(x \text{ unlocks } y)$
"There is a password such that there is a computer that the password unlocks"
"Some password unlocks some computer"

$\forall_{y\in Q}\forall_{x\in P}(x \text{ unlocks } y)$
"For all computers, and all passwords, each password unlocks each computer"
"Every computer can be unlocked by every password

3) Let x and y be even integers. Prove that the product xy is an even integer. (6 points)

Because x and y are even integers, we can write $x = 2k_1$ and $y = 2k_2$ for some integers k_1, k_2.

$$xy = (2k_1)(2k_2) = 2(2k_1k_2)$$

Here we see that indeed xy is even because it is written as 2 times some integer, namely $2k_1k_2$.

Two common mistakes:
(1) Using specific numbers such as $4 \cdot 6 = 24$. It is true that 24 is even, but if we do it like this we would need to verify for all infinitely many even numbers that their product is even.
(2) Saying what you want to do but not actually proving it: Yes xy is even with x and y are even, but we need to see why that is the case.
(3) Working with $\frac{x}{2}$ and $\frac{y}{2}$. When you divide out by 2, you lose the fact that the number is even, so it gets a lot harder to work with.