Name ___Solutions____ Discrete I, Quiz 4

1) Use induction to prove the equality below for all positive integers n.

$$\sum_{i=1}^{n} 6i = 3n(n+1)$$

BC:

$$\sum_{i=1}^{1} 6i = 6 \cdot 1 = 3 \cdot 2 = 3 \cdot 1 \cdot (1+1)$$

IH: Assume $\sum_{i=1}^{k} 6i = 3k(k+1)$ for some positive integer k.

IS: Now we show the "$k+1^{th}$ case"

$$\begin{aligned}\sum_{i=1}^{k+1} 6i &= 6 + 12 + 18 + \cdots + 6k + 6k(k+1) \\ &= [6 + 12 + 18 + \cdots + 6k] + 6(k+1) \\ &= [3k(k+1)] + 6(k+1) \\ &= 3k(k+1) + 6(k+1) \\ &= 3[k(k+1) + 2(k+1)] \\ &= 3(k+1)[k+2] \\ &= 3(k+1)(k+2)\end{aligned}$$

Therefore, for all positive integers n we have proven:

$$\sum_{i=1}^{n} 6i = 3n(n+1)$$

Some common mistakes have color-coded comments:

- Blue = Mathematical mistakes, such as:
  - Incorrect algebra
- Green = things where the mathematical grammar doesn't make sense, such as:
  - "i" outside of a summation
  - Calling a number "true" instead of referring to an equation.
  - Assuming a number instead of referring to an equation.
- Orange = things that should be true, but that weren't justified, such as:
  - Circular reasoning – writing down what you're trying to prove as if that were the proof.

Common logical error (In both the BC and IS): "Proving" something that is obvious, such as $6 = 6$ or $3k(k+1) = 3k(k+1)$.