Name __Solutions____ Discrete I, Quiz 10

1) What is the asymptotic runtime of the algorithm shown below?

```
for i from 0 to n-1
   "Line 2"
   for j from 0 to n-1
      "Line 4"
   for k from 0 to n-1
      "Line 6"
```

$O(n^2)$

2) Call your answer to the previous question $g(n)$. Justify your answer to the previous question by finding the constant multiple and point that it starts to apply: (Fill in the boxes; show and supporting work or derivation below)

$$f(n) \leq \boxed{2} \cdot g(n) \text{ whenever } n \geq \boxed{1}$$

The runtime is $n(n+n) = n^2 + n^2 \leq 2n^2$.

3) In the pseudocode below, "`Search_Database(m)`" is the function of interest, and runs in $O\big(h(m)\big)$ time. Everything else is trivial. What is the asymptotic growth rate of this algorithm? (Bonus Question)
(Technically to simplify this like I do below, we need to know that $h(m)$ is an increasing function. But this is a safe assumption because it measures the runtime of an algorithm.)

```
for i from 0 to n-1
   "Search_Database(i)"
   for j from 0 to n-1
      "Return_Records(j)"
      for k from i to j
         "Do_stuff(i,j,k)"
```

First note that we say that everything other than "`Search_Database(m)`" is trivial, so we can safely ignore lines 4 and 6. Now "`Search_Database(m)`" runs n times with inputs ranging from 0 to $n-1$. Hence we have the runtime $O\big(n \cdot h(n)\big)$ as derived below.

Plugging in each i we get a runtime of $O\big(h(1)\big) + O\big(h(2)\big) + \cdots + O\big(h(n-1)\big)$.
Now note that $O\big(h(i)\big)$ is $O\big(h(n)\big)$ because $i \leq n$ so $h(i) \leq h(n)$. Hence we get:

$$O\big(h(n)\big) + O\big(h(n)\big) + \cdots + O\big(h(n)\big) = O\big(n \cdot h(n)\big)$$