1) Let $A = \{1,2,3,4\}, B = \{a,b,c,d\}, C = \{1,2,a,b,5,6\}$. Find $(C \cap B) - A$. (-3/+3 points)

2) On the Venn Diagram below, shade the set $A^c \cap B$. (-3/+1 points)
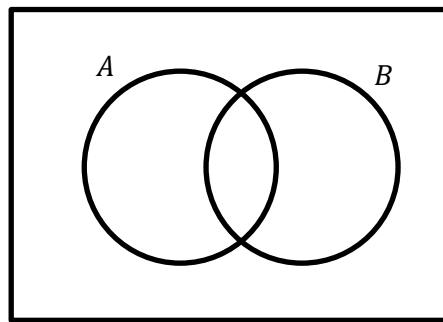


3) Write a truth table for the statement $(p \lor q) \land \neg p$ (-2/+2 points)

4) Suppose you have 11 dollars and 5 friends. You wish to give all the money to your friends. Justify the claim that at least one friend gets more than 2 dollars. (-1/+5 points)

5) Use a Venn Diagram to illustrate the statement "When Red Alert is activated, the shields go up". (-3/+1 points)

6) Determine whether or not the argument below is valid. Illustrate this on your Venn Diagram above. "When Red Alert is activated, the shields go up. The Enterprise is under Red Alert. Therefore the shields are up." (-2/+2 points)

7) Write a complete and grammatically correct sentence that has the form "$\forall_x(P(x))$" (-2/+4 points)

Determine whether each of the following is true or false. (-0.5/+0.5 points each)

8) $\forall_{x\in\mathbb{R}}(x^3 \geq 0)$

9) $\exists_{x\in\mathbb{R}}(x^3 \geq 0)$

10) $\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(xy = 1)$

11) $\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy = 1)$

12) $\forall_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy = 1)$

13) $\exists_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(xy = 1)$

14) For all rational numbers $x$ and $y$, prove that $x - y$ is rational. (-2/+4 points)

15) Prove that there exist integers $x, y$ and $z$ such that $xy = z^2$. (-5/+1 points)

16) Find $\mathcal{P}(\{a, b, c\})$. (-2/+2 points)

17) Write a complete and grammatically correct sentence that has the form "$p \lor q$". (-2/+2 points)

18) Prove the equality below for all integers $n \geq 1$. (-10/+10 points)

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

19) Show that every integer at least 6 can be written as a sum of 2s and 7s. That is, for every $n \geq 6$ there are integers $x$ and $y$ such that $n = 2x + 7y$. (-10/+10 points)

**Bonus Question**

20) Prove that $\sqrt{7}$ is irrational. (-0/+10 points)