Name _____ Discrete I, Test 2, Fall 2015

For all the problems on this page, use the function defined below. If you need more space, give your answer to the right and mark which problem it completes.

$$f: \{1,2,3\} \to \{0,1,2,3,4,5,6,7,8,9\}$$
$$x \to x^2$$

1) What is the domain of f? (-2/+1 points)

2) What is the codomain of f? (-2/+2 points)

3) What is the range of f? (-2/+2 points)

4) What is $f^{-1}(4)$? (-3/+1 points)

5) What is $f^{-1}(5)$? (-1/+2 points)

6) What is $f(2)$? (-3/+1 points)

7) Sketch a graph of f. (-1/+3 points)

8) Use an arrow diagram to illustrate f. (-1/+3 points)

9) Let S be the range of f, and a new function $g: \{1,2,3\} \to S$ is defined via $g(x) = f(x)$.
What is g^{-1}? Define it completely. (bonus question; -0/+5 points)

10) Define the numbers $c_0, c_1, c_2, c_3, c_4, \ldots$ via $c_0 = 1$ and $c_n = c_{\lfloor \frac{n}{3} \rfloor} + \frac{4}{3}$. Prove, using strong induction, that $c_n < 2n$ for all $n \geq 2$. (-10/+10 points)

For the problems on this page, consider the relation R on $\mathbb{Z}$ defined by xRy if and only if $x^3 = y^3$. Choose and complete THREE of these problems, and THREE only. If more answers are given, only the first THREE will be graded. (-5/+5 points each)

11) Is R reflexive? Justify your answer.
12) Is R symmetric? Justify your answer.
13) Is R antisymmetric? Justify your answer.
14) Is R transitive? Justify your answer.
15) Is R total? Justify your answer.
16) Is R irreflexive? Justify your answer.

Choose and complete ONE of these problems, and ONE only. If more answers are given, only the first ONE will be graded. (-10/+10 points)

17) Define a relation S on $\mathbb{Z}$ by xSy if and only if x divides y. Show that S is a partial order relation.

18) Show that the function f, below, is both one to one and onto.

$$f: \mathbb{R} \to \mathbb{R}$$
$$x \to 5x + 7$$