Name ____ Discrete I, Test 3, Fall 2015

For all the problems on this page, use the sequence $\{a_n\}_{n=1}^{\infty}$ defined below.

$$a_n = 2n + 1, \quad n = 1, 2, 3, 4, \ldots$$

1) Write out the first five terms of the sequence. (-3/+1 points)

2) Find $\sum_{n=2}^{5} a_n$. (-2/+2 points)

3) Reindex $\sum_{n=7}^{450} a_n$ so that it starts at $n = 0$. (Don't try to calculate it. Just reindex it) (-1/+3 points)

T or F 4) The sequence $\{a_n\}_{n=1}^{\infty}$ is increasing. (-0.5/+0.5 points)

T or F 5) The sequence $\{a_n\}_{n=1}^{\infty}$ is decreasing. (-0.5/+0.5 points)

T or F 6) The sequence $\{a_n\}_{n=1}^{\infty}$ is nonincreasing. (-0.5/+0.5 points)

T or F 7) The sequence $\{a_n\}_{n=1}^{\infty}$ is nondecreasing. (-0.5/+0.5 points)

For all the problems on this page, use the strings $r =$ "$aab123$" and $s =$ "$aabcda$" from the alphabet $X = \{a, b, c, d, e, f, 1,2,3\}$

8) What is the length of s? (-3/+1 points)

9) What is rs? (-1/+2 points)

T or F 10) "aab" is a substring of s. (-0.5/+0.5 points)

T or F 11) "aad" is a substring of s. (-0.5/+0.5 points)

T or F 12) "adc" is a substring of s. (-0.5/+0.5 points)

13) How many strings are there of length exactly 3 over X? (-2/+2 points)

14) List all strings that are in X^* but not in X+. (-1/+2 points)

For all the problems on this page, use the algorithm given below.

Input: n numbers given by $s_0, s_1, s_2, \dots, s_{n-1}$.
Output: 5, if the input includes 5, otherwise -1.

```
for i = 0 to n − 1
     if s_i == 5 then
         n = s_i − 3
         Return s_i
return −1
```

15) Trace through this algorithm with the input $6, 2, 7, 5, 3, 4$ by making a table of all 8 variables and showing if/how they change. (-5/+5 points)

T or F 16) The algorithm is both deterministic and finite. (-0.5/+0.5 points)

T or F 17) The algorithm is correct. (-0.5/+0.5 points)

18) Asymptotically (specifically big-Theta), how many times does “Do Stuff” run in this algorithm? (-5/+5 points)
(60% credit given if you use big-Oh instead of big-Theta)

```
for i from 0 to n-1
    "Do Stuff"
    for j from 0 to n-1
        for k from 0 to n-1
        "Do Stuff"
```

19) Asymptotically (specifically big-Theta), how many times does “Do Stuff” run in this algorithm? (-5/+5 points)
(60% credit given if you use big-Oh instead of big-Theta)

```
for i from 0 to n-1
    for j from 0 to i
        for k from 0 to i
        "Do Stuff"
```

20) Asymptotically (specifically big-Theta), how many times does “Do Stuff” run in this algorithm? (-5/+5 points)
(60% credit given if you use big-Oh instead of big-Theta)

```
for i from 0 to 5
    for j from 0 to n
        "Do Stuff"
```

21) An air traffic control office is set up to submit flight logistics problems to a cloud-based server. The cloud service has three options for algorithms. All correctly perform the calculations the air traffic control office requires. Their asymptotic runtimes are:

(a) $O(n^3)$
(b) $\Theta(n^3)$
(c) $\Omega(n^3)$

Which algorithm, (a), (b), or (c), should the air traffic control office choose? Why? (-5/+5 points)

22) The same air traffic control office decided to reject your recommendation and requested more information from the cloud service. You now have access to both the runtime and the cost of each algorithm. As long as the algorithm runs in a reasonable amount of time, no planes will crash. That is your first priority, to prevent planes from crashing. However, your second priority is to save money.

(a) $O(n^3)$ costs $10,000 per month
(b) $\Theta(n^3)$ costs $8,000 per month
(c) $\Omega(n^3)$ costs $6,000 per month

Now which algorithm, (a), (b), or (c), should you choose? Why? (-5/+5 points)

T or F 23) n^3 is $\Theta(n^2)$
T or F 24) n^3 is $O(n^2)$
T or F 25) n^3 is $\Omega(n^2)$
T or F 26) n^3 is $\Theta(n^3)$
T or F 27) n^3 is $O(n^3)$
T or F 28) n^3 is $\Omega(n^3)$
T or F 29) n^3 is $\Theta(n^4)$
T or F 30) n^3 is $O(n^4)$
T or F 31) n^3 is $\Omega(n^4)$
T or F 32) $n^3 + n^2$ is $\Theta(n^3)$
(-0.25/+0.25 points each)

Bonus Question

33) You work for the technology department of a growing petroleum distribution company. You currently use a $\Theta(n^6)$ algorithm to compute routing operations for the various products you move. For a cost of $80,000 to the company your department can upgrade this program to run in $O(n^3)$ time. Your boss, Robert, is skeptical if spending that much money would actually be worth it. In a couple grammatically correct sentences, write a compelling memo to Robert to try to convince him that upgrading this software would be beneficial to the company. (-0/+5 points)