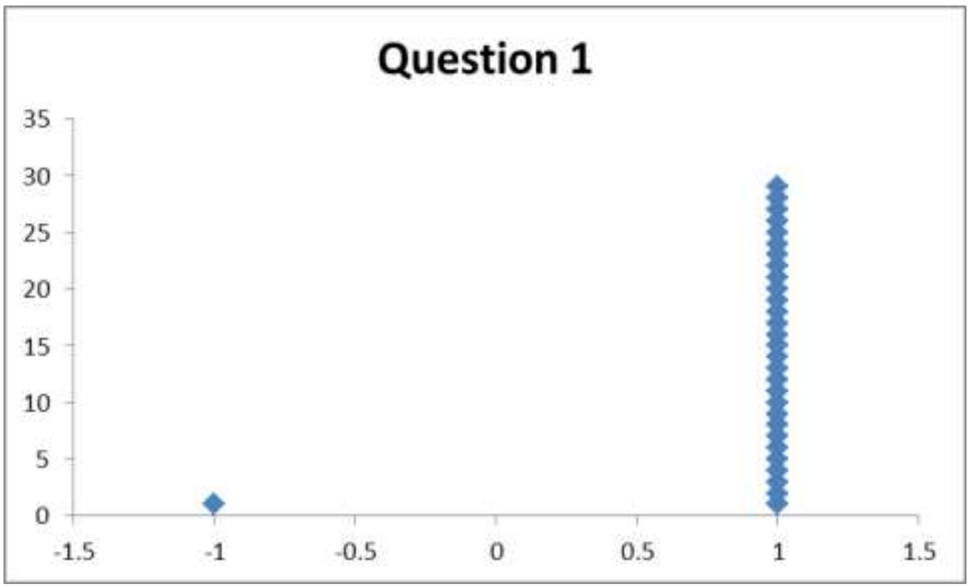
Name ___Solutions____ Discrete I, Test 3, Fall 2015

For all the problems on this page, use the sequence $\{a_n\}_{n=1}^{\infty}$ defined below.

$$a_n = 2n + 1, \quad n = 1, 2, 3, 4, \ldots$$

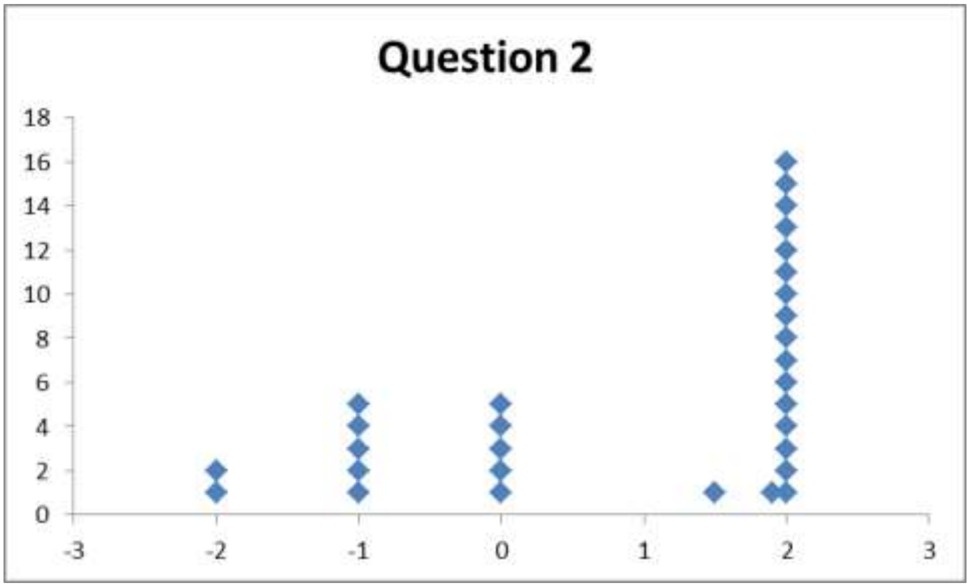
1) Write out the first five terms of the sequence. (-3/+1 points)

$a_1 = 2 \cdot 1 + 1 = 3$
$a_2 = 2 \cdot 2 + 1 = 5$
$a_3 = 7$
$a_4 = 9$
$a_5 = 11$



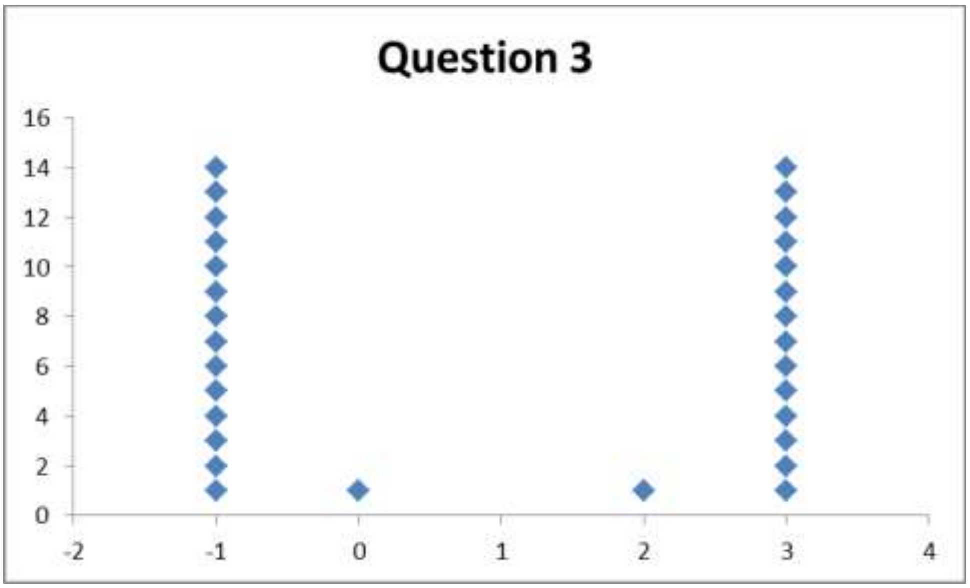
2) Find $\sum_{n=2}^{5} a_n$. (-2/+2 points)

$a_2 + a_3 + a_4 + a_5 = 5 + 7 + 9 + 11 = 32$



3) Reindex $\sum_{n=7}^{450} a_n$ so that it starts at $n = 0$. (Don't try to calculate it. Just reindex it) (-1/+3 points)

$$\sum_{n=7}^{450} a_n = \sum_{n=0}^{443} a_{n+7}$$

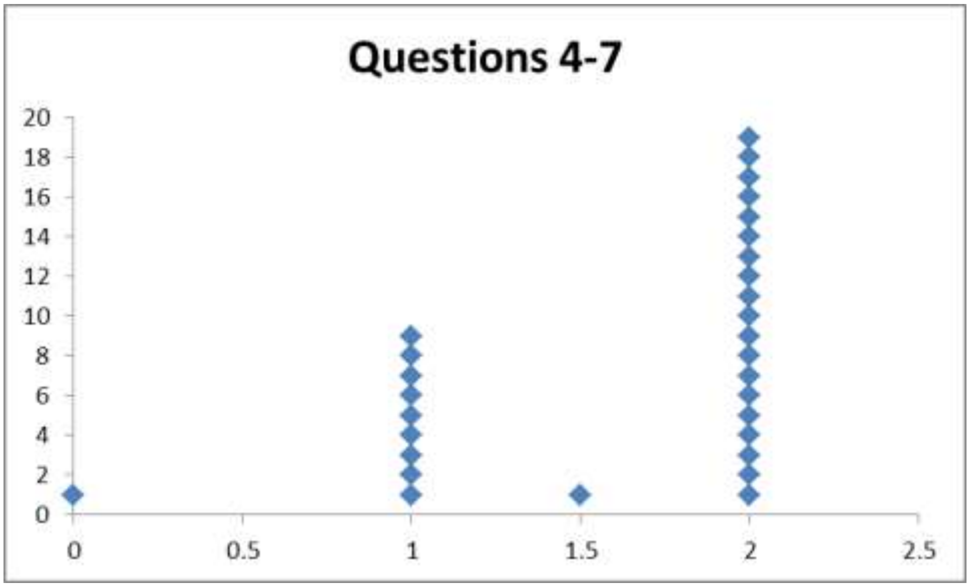


T or F 4) The sequence $\{a_n\}_{n=1}^{\infty}$ is increasing. (-0.5/+0.5 points)

T or F 5) The sequence $\{a_n\}_{n=1}^{\infty}$ is decreasing. (-0.5/+0.5 points)

T or F 6) The sequence $\{a_n\}_{n=1}^{\infty}$ is nonincreasing. (-0.5/+0.5 points)

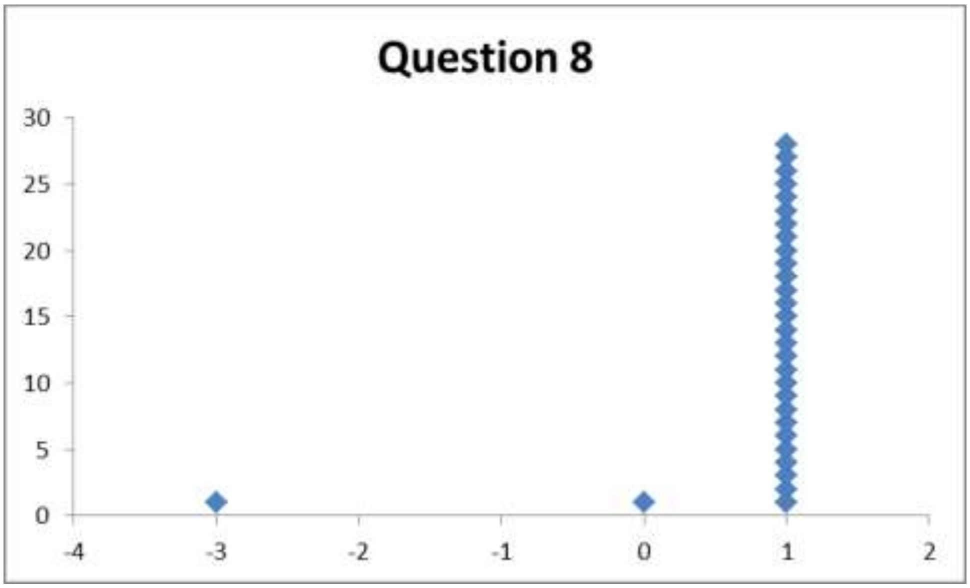
T or F 7) The sequence $\{a_n\}_{n=1}^{\infty}$ is nondecreasing. (-0.5/+0.5 points)



For all the problems on this page, use the strings $r =$ "$aab123$" and $s =$ "$aabcda$" from the alphabet $X = \{a, b, c, d, e, f, 1,2,3\}$

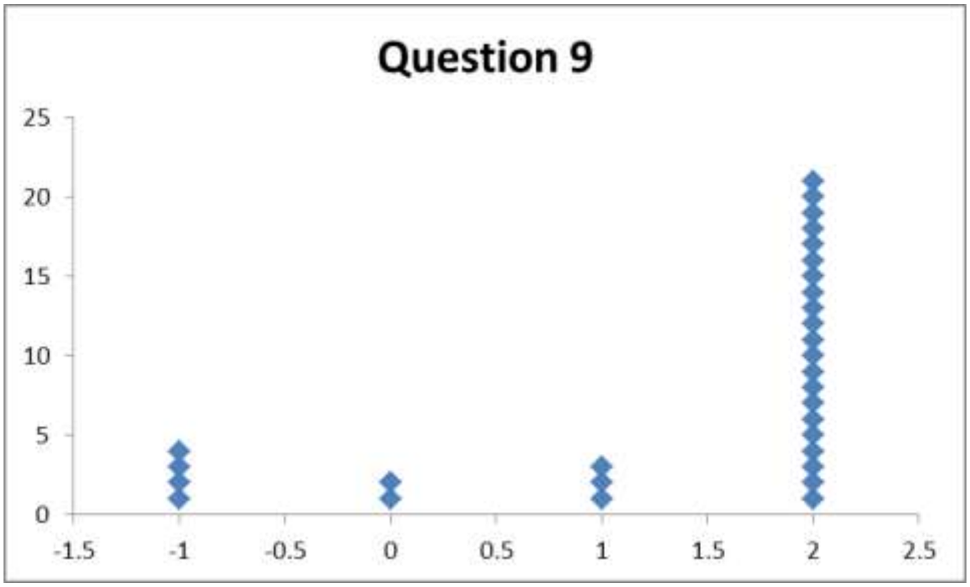
8) What is the length of s? (-3/+1 points)

6



9) What is rs? (-1/+2 points)

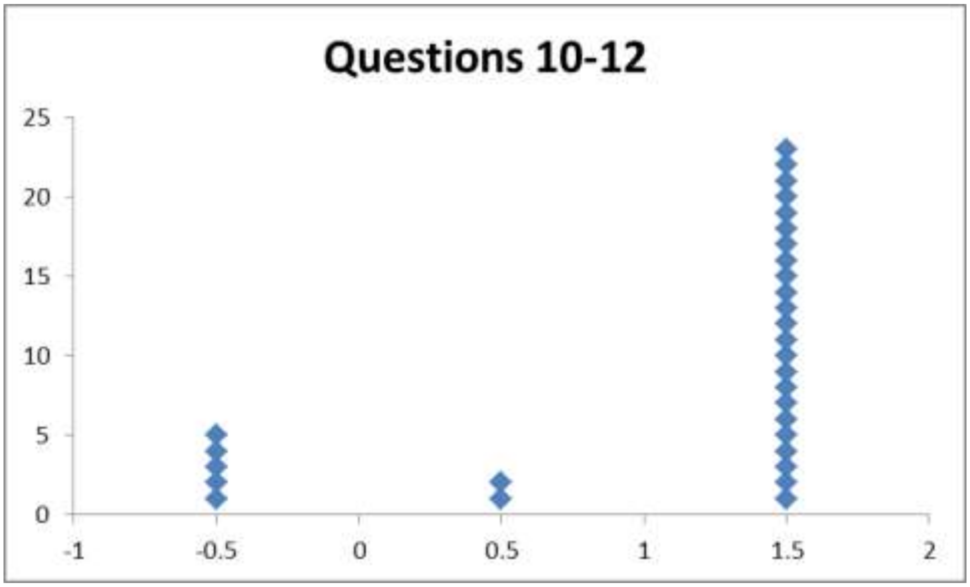
$aab123aabcda$



T or F 10) "aab" is a substring of s. (-0.5/+0.5 points)

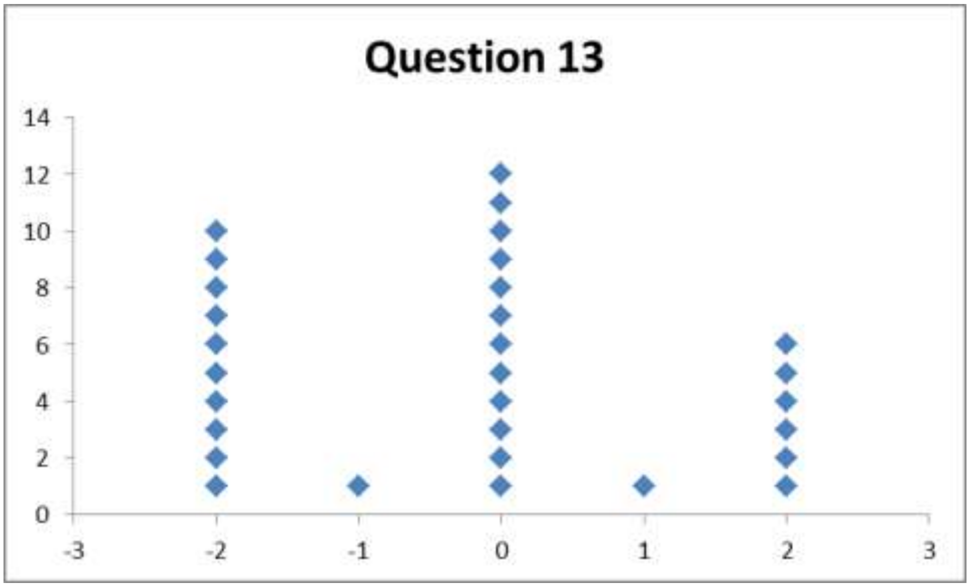
T or F 11) "aad" is a substring of s. (-0.5/+0.5 points)

T or F 12) "adc" is a substring of s. (-0.5/+0.5 points)



13) How many strings are there of length exactly 3 over X? (-2/+2 points)

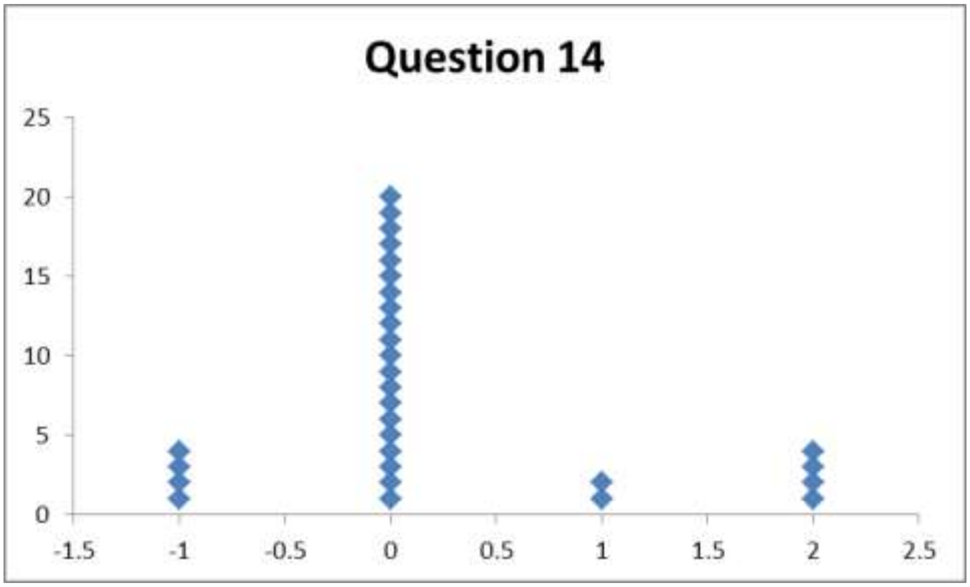
9^3



14) List all strings that are in X^* but not in $X+$. (-1/+2 points)

The empty string, “”.

Note that this is not the same as the empty set {} or Ø.



For all the problems on this page, use the algorithm given below.

Input: n numbers given by $s_0, s_1, s_2, \dots, s_{n-1}$.
Output: 5, if the input includes 5, otherwise -1.

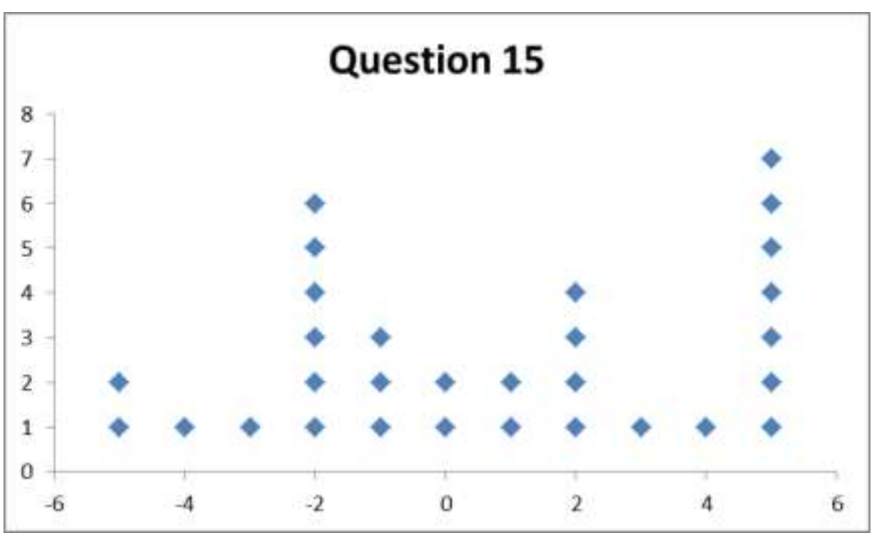
```
for i = 0 to n − 1
   if s_i == 5 then
      n = s_i − 3
      return s_i
   else continue
return −1
```

15) Trace through this algorithm with the input $6, 2, 7, 5, 3, 4$ by making a table of all 8 variables and showing if/how they change. (-5/+5 points)

s_0	s_1	s_2	s_3	s_4	s_5	i	n
6	2	7	5	3	4	0	5
						1	2
						2	
						3	

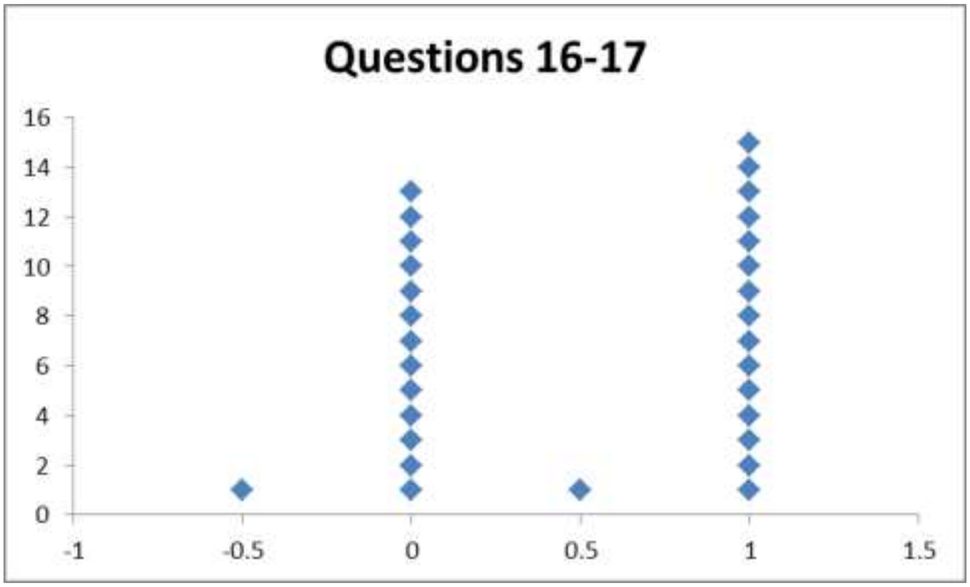
Clarification on various common missunderstandings:

- I told you exactly what to do, *make a table of all* 8 *variables.*
- The input consists of n integers. n could be any positive whole number, but with the input I gave you, $n = 6$.
- You could think of s as an array and s_i as $s[i]$ if you like, although that is only one possible implementation of this psuedocode.
- When the algorithm returns, it stops running, so with the given input i never has a chance to get to 4.



T or F 16) The algorithm is both deterministic and finite. (-0.5/+0.5 points)

T or F 17) The algorithm is correct. (-0.5/+0.5 points)

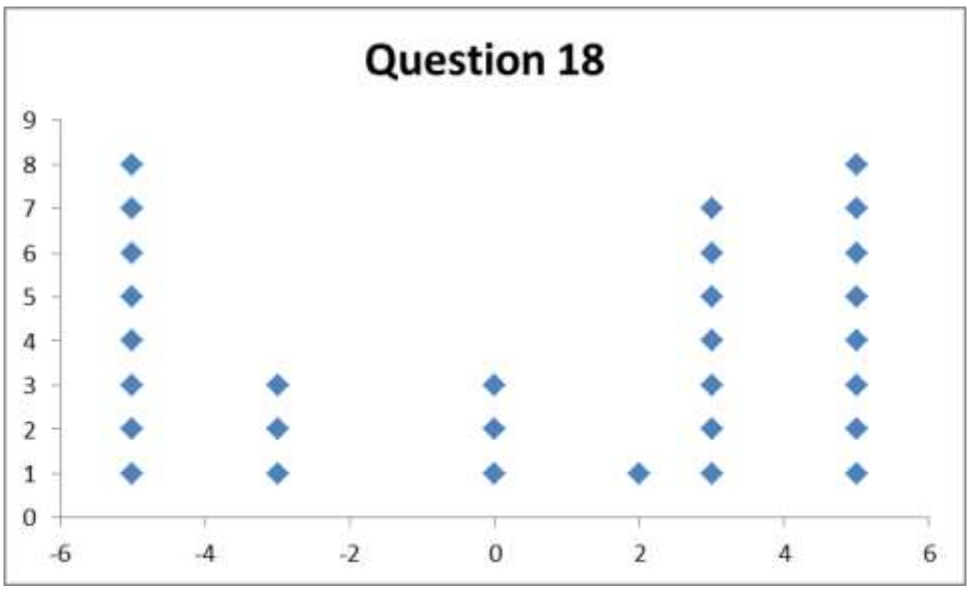


18) Asymptotically (specifically big-Theta), how many times does `"Do Stuff"` run in this algorithm? (-5/+5 points)
(60% credit given if you use big-Oh instead of big-Theta)

```
for i from 0 to n-1
    "Do Stuff"
    for j from 0 to n-1
        for k from 0 to n-1
        "Do Stuff"
```

$\Theta(n^3)$

Specifically, it is exactly $n^3 + n$ which is $\Theta(n^3)$

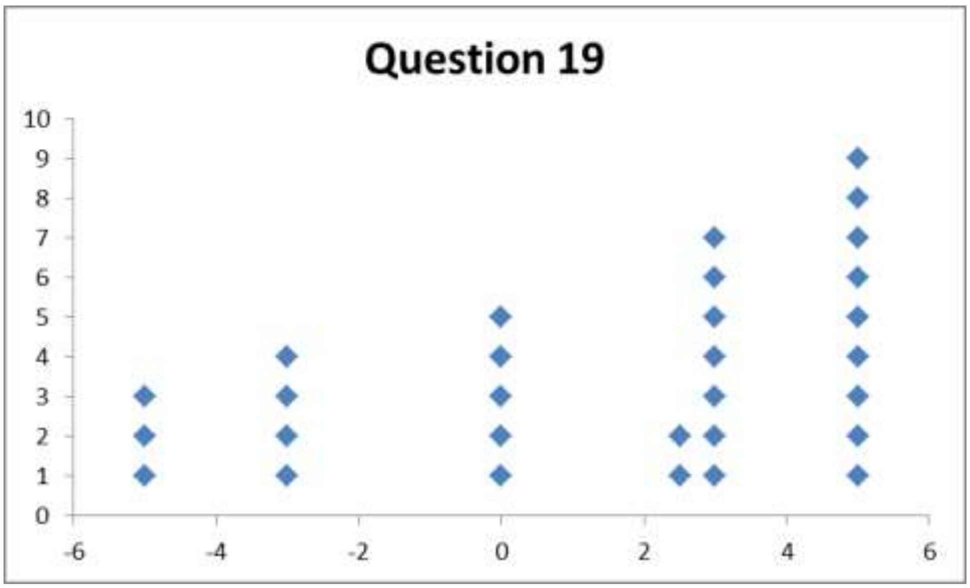


19) Asymptotically (specifically big-Theta), how many times does `"Do Stuff"` run in this algorithm? (-5/+5 points)
(60% credit given if you use big-Oh instead of big-Theta)

```
for i from 0 to n-1
    for j from 0 to i
        for k from 0 to i
        "Do Stuff"
```

$\Theta(n^3)$

Specifically it is $1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ which is $\Theta(n^3)$

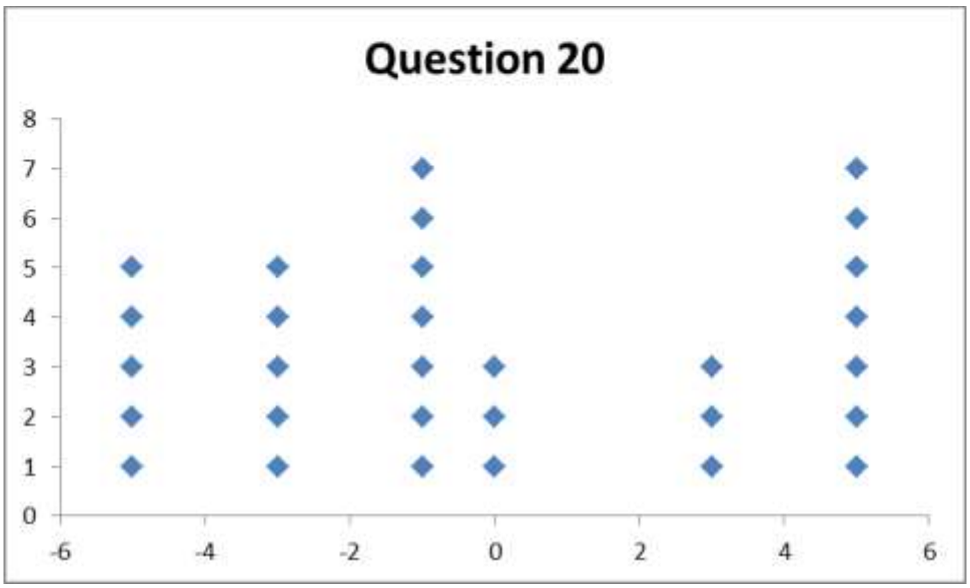


20) Asymptotically (specifically big-Theta), how many times does `"Do Stuff"` run in this algorithm? (-5/+5 points)
(60% credit given if you use big-Oh instead of big-Theta)

```
for i from 0 to 5
    for j from 0 to n
        "Do Stuff"
```

$\Theta(n)$

Specifically it is $6n$ which is $\Theta(n)$. Watch your loops! That first loop doesn't depend on n at all. We could have just copied and pasted the code 6 times!

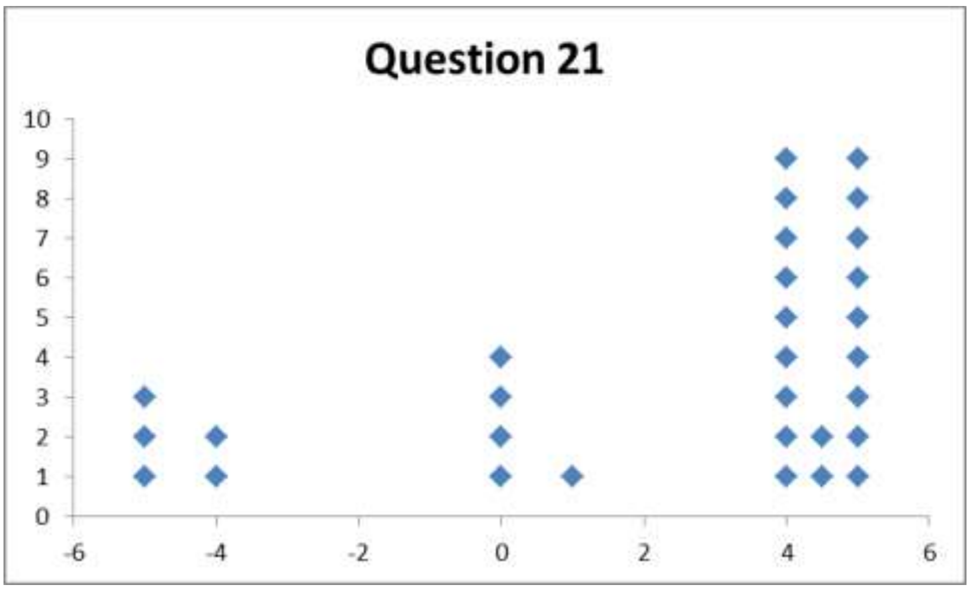


21) An air traffic control office is set up to submit flight logistics problems to a cloud-based server. The cloud service has three options for algorithms. All correctly perform the calculations the air traffic control office requires. Their asymptotic runtimes are:

(a) $O(n^3)$
(b) $\Theta(n^3)$
(c) $\Omega(n^3)$

Which algorithm, (a), (b), or (c), should the air traffic control office choose? Why? (-5/+5 points)

An algorithm that runs in $O(n^3)$ time has the potential to be faster, so all things equal let's take the chance that it is!



22) The same air traffic control office decided to reject your recommendation and requested more information from the cloud service. You now have access to both the runtime and the cost of each algorithm. As long as the algorithm runs in a reasonable amount of time, no planes will crash. That is your first priority, to prevent planes from crashing. However, your second priority is to save money.

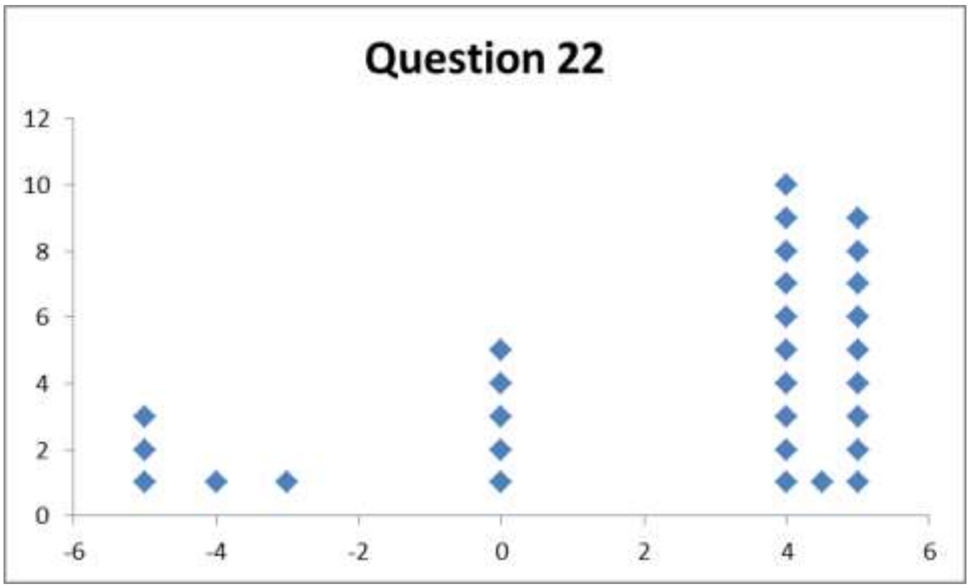
(a) $O(n^3)$ costs $10,000 per month

(b) $\Theta(n^3)$ costs $8,000 per month

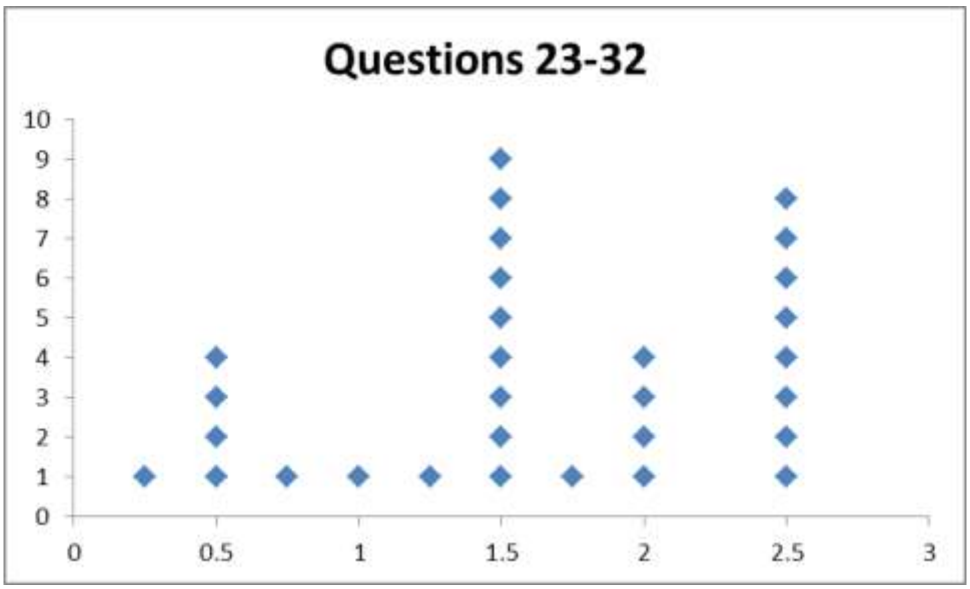
(c) $\Omega(n^3)$ costs $6,000 per month

Now which algorithm, (a), (b), or (c), should you choose? Why? (-5/+5 points)

Now all things are not equal. An algorithm that runs in $O(n^3)$ could very well actually run in $\Theta(n^3)$ so let's not take the chance that (a) is faster, and instead take the $2,000 per month savings.



T or F 23) n^3 is $\Theta(n^2)$
T or F 24) n^3 is $O(n^2)$
T or F 25) n^3 is $\Omega(n^2)$
T or F 26) n^3 is $\Theta(n^3)$
T or F 27) n^3 is $O(n^3)$
T or F 28) n^3 is $\Omega(n^3)$
T or F 29) n^3 is $\Theta(n^4)$
T or F 30) n^3 is $O(n^4)$
T or F 31) n^3 is $\Omega(n^4)$
T or F 32) $n^3 + n^2$ is $\Theta(n^3)$
(-0.25/+0.25 points each)



Bonus Question

33) You work for the technology department of a growing petroleum distribution company. You currently use a $\Theta(n^6)$ algorithm to compute routing operations for the various products you move. For a cost of $80,000 to the company your department can upgrade this program to run in $O(n^3)$ time. Your boss, Robert, is skeptical if spending that much money would actually be worth it. In a couple grammatically correct sentences, write a compelling memo to Robert to try to convince him that upgrading this software would be beneficial to the company. (-0/+5 points)

If the current $\Theta(n^6)$ is working, there isn't much motivation to change it, especially if it costs $80,000 to do so. However, the key piece of information is that the company is *growing*. As it grows, n^6 far outpaces n^3. So at some point the company will be forced to upgrade, and it is best to do that now before the current algorithm becomes ineffective. As such, an example of a memo would be as given below.

Robert,

As you know, we currently use a routing algorithm that runs in $\Theta(n^6)$ time. It's worked for us to date, but as we grow it will eventually fail to compute the necessary routing operations on time. This could be a disaster should that ever happen, so we should upgrade this program now. At a cost of $80,000 we can upgrade to an algorithm that runs in only $O(n^3)$. This will allow the company to grow much larger than our current potential.

IT department