

Codename _____ Transitions, Test 1

(Do not put your name on the test; write your name and codename on the code sheet)

1) Graham's number, g_{64} , is a very very large real number. Consider the statement $\exists_{x \in \mathbb{R}} (x > g_{64})$.

a) Write this statement as an English sentence.

b) Prove this statement.

2) Consider the statement $\forall_{x < 0} (x^2 > 0)$.

a) Write this statement as an English sentence.

b) Prove this statement. (Hint: if $y < 0$, then $y = -|y|$)

3) Consider the statement "If a set is a subset of the empty set, then that set is empty."

a) Write this statement in mathematical notation.

b) Prove this statement.

4) Define a singleton to be a set with a single element, such as $\{1\}$. Consider the statement "The cross product of a set with a singleton is the same size as the original set".

a) Write this statement in mathematical notation.

b) Give a sketch of the proof of the statement.

c) Which of the following are singletons? Circle them.

$\{1,2\}$, $\{\{1,2\}\}$, $\{1\}$, $\{2\}$, $\{\{1\}\}$, $\{\{2\}\}$, $\{\emptyset\}$, $\{\{\emptyset\}\}$, $\{\{\}\}$

5) Show the following theorem for all sets A :

$$A = \bigcup_{B \in \mathcal{P}(A)} B$$

6) Show the following theorem for all sets $A, B,$ and $C.$

$$(A - B) - C = A - (B \cup C)$$

7) Show the following theorem for all statements P and Q :

$$(P \Leftrightarrow Q) \Rightarrow (\sim P \vee Q)$$

8) Show the following theorem:

$$\forall x > 0 \exists y > 0 \forall z > 0 [(z < y) \Rightarrow (3z < x)]$$