Codename ____ Transitions, Test 2

(Do not put your name on the test; write your name and codename on the code sheet)

1) Show that for all $n \in \mathbb{Z}_{\geq 1}$:

$$\sum_{i=1}^{n} i \cdot 2^i = 2 + (n-1)2^{n+1}$$

2) Let A be the set of all people. Choose ONE of the following relations and show that it is an equivalence relation:

R_1 is the relation on A such that xRy if and only if x and y have the same shoe size.

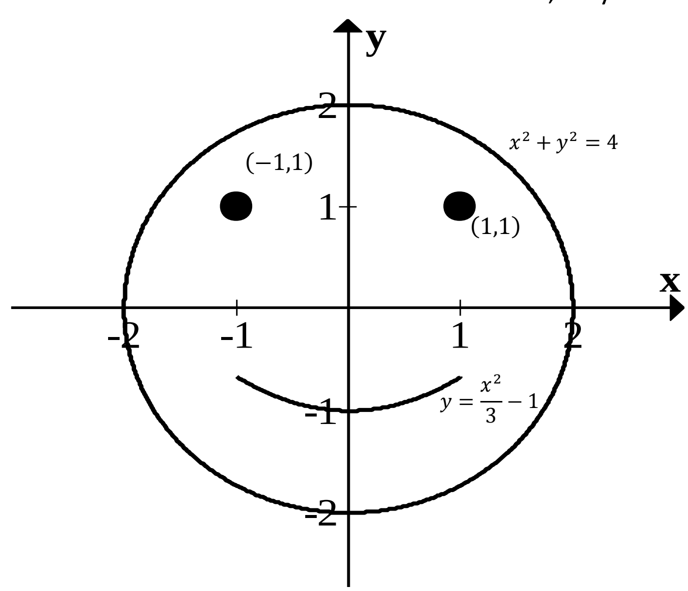
R_2 is the relation on A such that xRy if and only if x and y are either both male, or both female.

3) Let $f\colon \mathbb{Z} \to \mathbb{R}$ be the relation given by $f(x) = \sqrt{x}$ when possible. Sketch a graph of f then prove or disprove that f is a function.

4) Let $f\colon \mathbb{R} \to \mathbb{C}$ be the relation given by $f(x) = \sqrt{x}$. Prove or disprove that f is a function.

5) Let $f\colon \mathbb{R} \to \mathbb{C}$ be the relation given by $f(x) = \pm\sqrt{x}$. Prove or disprove that f is a function.

6) Let R be the graph of the entire smiley face below. Is R a relation? If so what is it as a set? If not, why not?



7) Let $f: \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = 2x + 4$. Sketch a graph of f and prove or disprove that f is one-to-one.

8) Let $f: \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = 2x + 4$. Prove or disprove that f is onto.

9) Let A be a set, and define a binary relation $\rtimes$ on A. (For example, $\mathbb{R}$ and addition satisfy this). Now suppose that $\rtimes$ is actually associative: $a \rtimes b \rtimes c$ is unambiguous in that $(a \rtimes b) \rtimes c = a \rtimes (b \rtimes c)$ for all $a, b, c \in A$. Sketch a proof of the fact that for any $n \in \mathbb{Z}_{\geq 3}$:

$$a_1 \rtimes a_2 \rtimes \cdots \rtimes a_n \text{ is unambiguous.}$$

Consider the following function diagram:

$$\begin{array}{ccccc} \mathbb{R} & \xrightarrow{f_1} & \mathbb{R} & \xrightarrow{f_2} & \mathbb{R} \\ f_3 \downarrow & \circlearrowleft & \downarrow f_4 & & \downarrow f_5 \\ \mathbb{Q} & \xrightarrow[f_6]{} & \mathbb{Z} & \xrightarrow[f_7]{} & \mathbb{R} \end{array}$$

10) Find functions $f_1, f_2, \dots, f_7$ such that the diagram is commutative in the left square, but not in the right square.

$f_1(x) =$

$f_2(x) =$

$f_3(x) =$

$f_4(x) =$

$f_5(x) =$

$f_6(x) =$

$f_7(x) =$

11) Is it true that $f_4 \circ f_1 = f_6 \circ f_3$? Why?

12) Is it true that $f_5 \circ f_2 = f_7 \circ f_4$? Why?

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Let A be the set of all monomials involving the variables x and or y. (A monomial is a term consisting of variables to nonnegative integer powers, all multiplied by each other).
For example, the following are all such monomials:

$$x \qquad x^6 \qquad xy^2 \qquad x^{240}y$$

As a nonexample, the following are not elements of A:

$$2x \qquad xy^2z^7t \quad x^{-1}y \qquad x^{2.5}$$

Define the total degree, d, of a monomial as the sum of the exponents.
For example $d(xy^7) = 8$ while $d(x^4y^2) = 6$.

Finally, for monomials a_1 and a_2, define the relation $\prec$ on A via $a_1 \prec a_2$ if and only if one of the following is satisfied:
$d(a_1) < d(a_2)$
OR
$d(a_1) = d(a_2)$ and the degree of x in a_1 is less than the degree of x in a_2.

As one last step, define $\preccurlyeq$ as " $\prec$ or $=$ ".

13) Fill in each of the following boxes with either $\preccurlyeq$ or $\succcurlyeq$:

$$x^2y^6 \,\square\, x^3y^6$$
$$x^2y^6 \,\square\, x^3y^5$$
$$x^2y^6 \,\square\, x^1y^7$$

Prove or disprove each of the following: (Use the back page and clearly label each problem)
14) $\preccurlyeq$ is reflexive.
15) $\preccurlyeq$ is symmetric.
16) $\preccurlyeq$ is antisymmetric.
17) $\preccurlyeq$ is transitive.
18) All elements of A are comparable under $\preccurlyeq$.
19) $\preccurlyeq$ is an equivalence relation.
20) $\preccurlyeq$ is a total ordering.