Name ___Solutions____ Transitions, Quiz 4

1) Solve $4x = 3$ mod 99.

$$4^{-1} \cdot 4x = 4^{-1} \cdot 3$$
$$x = 4^{-1} \cdot 3$$

What is 4^{-1}? We're looking for a number that gives 1 mod 99.
think
think
ah-ha!
$4 \cdot 25 = 100 = 1$ mod 99.

Then we get:

$$x = 4^{-1} \cdot 3 = 25 \cdot 3 = 75 \mod 99$$

Consider the equivalence relation on $\mathbb{R}^2$ that identifies points that are the same distance from the origin with each other. For instance, $(5,0), (0,5), (-5,0), (0,-5), (3,4), (4,3), (-3,4)$, among many other points should all be equivalent.

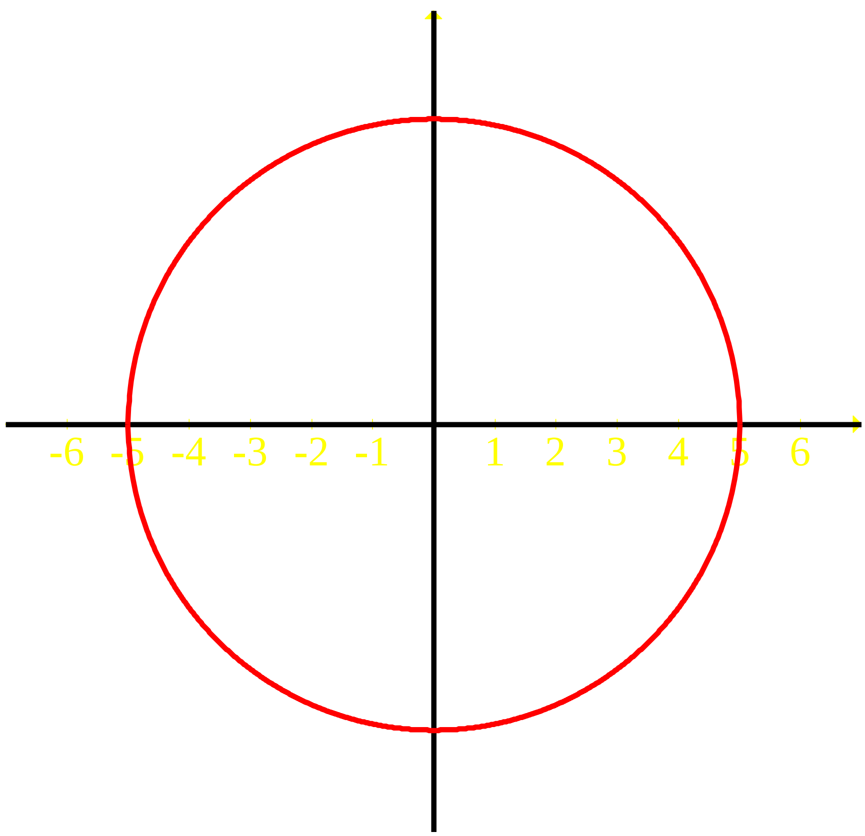
2a) Write down the equivalence relation as a set.

$\{((a,b),(c,d)) : a^2 + b^2 = c^2 + d^2\}$

2b) Describe the equivalence relation as an "iff" statement.

$(a,b)R(c,d)$ iff $a^2 + b^2 = c^2 + d^2$

2c) Sketch a graph of the equivalence class of $(0,5)$.



3) Consider the following total order relation on $\mathbb{Z}_{>0}$. We're going to split the positive integers into two groups: All even integers come before all odd integers. Within each group, use the standard ordering. For example, if we call this relation "$\preccurlyeq$" then we see that $2 \preccurlyeq 6, 3 \preccurlyeq 5$, and $8 \preccurlyeq 7$.

Provide a sketch a proof that "$\preccurlyeq$" is a total order relation.

Reflexive:
Let x be a positive integer. Then if x is even, $x \leq x$ so $x \preccurlyeq x$. Similarly if x is odd, $x \leq x$ so $x \preccurlyeq x$.

Antisymmetric:
Let x and y be positive integers and assume that $x \preccurlyeq y$ and $y \preccurlyeq x$.
If x and y are both even or both odd, then we have $x \leq y$ and $y \leq x$. The fact that $\leq$ is antisymmetric then tells us that $x = y$.
If, however, x and y are of different parity then something went horribly ary: we could not possibly have had both $x \preccurlyeq y$ and $y \preccurlyeq x$ because the even one must come first!

Transitive:
Let x, y and z be positive integers and assume that $x \preccurlyeq y$ and $y \preccurlyeq z$.
First note that if z is even, then y must also as well. Hence also x is even, so all three are even. In this case the fact that $\leq$ is transitive tells us that $x \leq z$ and so $x \preccurlyeq z$.
If z is not even, we will consider y. If y is even, then x must be as well. Then $x \preccurlyeq z$ because even numbers come first.
If both z and y are odd, then we will consider x. If x is even, $x \preccurlyeq z$ because even numbers come first. If x is odd, then by the transitivty of $\leq$ we get that $x \leq z$ and so $x \preccurlyeq z$.

Total:
Let x and y be positive integers. If x and y are of the same parity, the fact that $\leq$ is total lets us compare them. If x and y are not of the same parity, then one of them is even and that one comes first.