Codename ____ Transitions, Test 2

(Do not put your name on the test; write your name and codename on the code sheet)

Define $\mathbb{Z}_5 = \{0,1,2,3,4\}$ to be the integers mod 5.

1) What does it mean for a function $f\colon \mathbb{Z}_5 \to \mathbb{Z}_5$ to be one-to-one? (25 points)

2) Prove that the function below is one-to-one. (100 points. Be sure not to skip steps)

$$\begin{aligned} f\colon \mathbb{Z}_5 &\to \mathbb{Z}_5 \\ x &\mapsto 2x \end{aligned}$$

3) What is $\mathbb{R}^2$? Express it as a set. (10 points)

4) What does it mean for a function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ to be onto? (25 points)

5) Prove that the function below is onto. (100 points)

$$
\begin{gathered}
f\colon \mathbb{R}^2 \to \mathbb{R}^2 \\
(x, y) \mapsto (x, 2y)
\end{gathered}
$$

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6) Prove the following equality holds true for all $n \in \mathbb{Z}_{\geq 0}$. (100 points)

$$\sum_{i=0}^{n} (2i+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

A set S is called well-ordered if every subset of S has a smallest element. That is, every single subset of S has a smallest element.

7) Let S be a well-ordered set. Use the fact that S is well-ordered to construct a partial ordering on S. (50 points)

8) In fact the relation you constructed is a partial order relation. We won't prove all of this though, just part of it. Prove that your relation is antisymmetric (100 points)

Codename ____ Transitions, Sheet 3

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9) Sketch a graph the function below. (50 points)

$$f: [-3, 6] \cup \mathbb{Z}_{\geq 7} \to \mathbb{R}$$
$$x \mapsto x^2$$

10) What is the domain and range of $f \circ g \circ h$? Some properties of f, g, and h are given below. (50 points)

f maps from A onto B, such that the range of g maps to C and the rest of A maps to D.

g maps from E into A.

h maps from F onto E.

11) Complete ONE of the problems below. (100 points)

A) Define relation R on $\mathbb{R}^2$ by identifying points that are the same distance from the origin with each other. Prove that R is an equivalence relation.
B) Define a relation S on the set of all monomials in variables x and y via $x^a y^b S x^c y^d$ iff $a = c$ and $b \leq d$. Prove that S is a partial order relation.