Name ____ Transitions, Quiz 1

Consider the statement "there is an irrational number in which all its positive integer multiples are also irrational."

1) Write this statement in mathematical notation.

$$\exists_{x\in\mathbb{R}-\mathbb{Q}}\forall_{y\in\mathbb{Z}_{>0}}(xy\in\mathbb{R}-\mathbb{Q})$$

2) Prove the statement.

Choose $x=\pi$. Note that π is irrational.
Let $y\in\mathbb{Z}_{>0}$.
We will show that $xy=y\pi$ is irrational. Suppose for the purpose of later contradiction that $y\pi$ is rational. Then there are $a,b\in\mathbb{Z}$ such that $y\pi=\frac{a}{b}$.
Rearranging this we obtain $\pi=\frac{a}{yb}$, which says that π is rational. This is a contradiction, so $xy=y\pi$ must be irrational.
Therefore there is an irrational number, π, in which all its positive integer multiples are also irrational. $\square$