

Note that all of these have multiple correct answers, especially on problems 4-6 there are many, many ways of phrasing these statements.

For questions 1-3, write each statement in mathematical notation.

1) Every dog has a collar which is blue.

$$\forall_{\text{dogs}} \exists_{\text{collar}} (\text{Said color is blue})$$

2) There is a number for which the product with every real number is zero.

$$\exists_{x \in \mathbb{C}} \forall_{y \in \mathbb{R}} (xy = 0)$$

Common issue: which comes first?

3) If you go swimming in January, then all the polar bears will laugh.

$$\text{You go swimming in January} \Rightarrow \forall_{\text{polar bears } p} (p \text{ will laugh})$$

For questions 4-6, write each statement in English.

$$4) \forall_{x \in \mathbb{R}} \forall_{y \in \mathbb{Q}} \forall_{z \in \mathbb{Z}} (xyz + 2 \in \mathbb{C})$$

For all real numbers, all rational numbers, and all integers, the product of all three plus two is a complex number.

$$5) a \in \mathbb{R} \wedge b \in \mathbb{R} \Rightarrow (a + bi)(a - bi) \in \mathbb{R}$$

If a and b are both real numbers, then $(a + bi)(a - bi)$ is also real.

$$6) \forall_{\varepsilon > 0} \exists_{N \in \mathbb{N}} (n \geq N \Rightarrow |a_n - L| < \varepsilon)$$

For all positive epsilon, there is a natural number N in which $n \geq N$ implies $|a_n - L|$ is less than epsilon.