Name _ Solutions____ Transitions, Quiz 3

Define the Harmonic numbers, H_n, as the summation from 1 to $\frac{1}{n}$. More specifically:

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}$$

Prove that for all $n \in \mathbb{Z}_{\geq 1}$,

$$\sum_{i=1}^{n} H_i = (n+1)H_n - n$$

Want we to prove:

$$\forall_{n \in \mathbb{Z}_{\geq 1}} \left(\sum_{i=1}^{n} H_i = (n+1)H_n - n \right)$$

Base Case:

$$\sum_{i=1}^{1} H_i = H_1 = 1 = (1+1) \cdot 1 - 1 = (1+1)H_1 - 1$$

Induction Hypothesis: Assume for some $k \in \mathbb{Z}_{\geq 1}$:

$$\sum_{i=1}^{k} H_i = (k+1)H_k - k$$

Induction Step:

We need to prove:

$$\sum_{i=1}^{k+1} H_i = ((k+1)+1)H_{k+1} - (k+1)$$

We to this be plugging in the induction hypothesis and simplifying:

$$\sum_{i=1}^{k+1} H_i = \sum_{i=1}^{k} H_i + H_{k+1} = (k+1)H_k - k + H_{k+1}$$

$$= (k+1)\left(H_{k+1} - \frac{1}{k+1}\right) - k + H_{k+1}$$

$$= (k+2)(H_{k+1}) - \frac{k+1}{k+1} - k$$

$$= (k+2)(H_{k+1}) - 1 - k$$

$$= (k+2)(H_{k+1}) - (k+1)$$

□