Name ______Solutions____ Transitions, Quiz 4

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BC: Consider $n = 0$:

$$\sum_{i=0}^{0} 2^i = 2^0 = 1 = 2 - 1 = 2^{0+1} - 1$$

IH: Assume that for some $k \in \mathbb{Z}_{\geq 0}$:

$$\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$$

IS: Here we want prove the "$k + 1$ case":

$$\sum_{i=0}^{k+1} 2^i = 2^{k+2} - 1$$

To do this we'll start with the LHS and break up the summation:

$$\sum_{i=0}^{k+1} 2^i = \sum_{i=0}^{k} 2^i + 2^{k+1}$$

$$= 2^{k+1} - 1 + 2^{k+1}$$

$$= 2 \cdot 2^{k+1} - 1$$

$= 2^{k+2} - 1$W

This proves the inductive step, and so we know that for all $n \in \mathbb{Z}_{\geq 0}$:

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

□