Name ___Solutions____ Transitions, Quiz 5

Define a relation R on the integers via:

aRb iff a and b have the same digit in the 10's place.

1) Give five examples of pairs of numbers that are related.

$42R40$, $7R(-3)$, $24R123$, $2202R32407$, $16R112$, $4R4$

2) Give five examples of pairs of numbers that are not related.

$42 \not R 50$, $7 \not R (-13)$, $24 \not R 143$, $2202 \not R 32417$, $16 \not R 162$, $4 \not R 14$

3) Prove that R is an equivalence relation.

Reflexive:
Let x be an integer. Then all of the digits of x are the same as the digits of itself, in particulars in the 10's place. Hence R is reflexive.

Symmetric:
Let x and y be integers such that xRy. That means that x and y have the same digit in the 10's place, so clearly y and x do as well. That is, yRx.

Transitive:
Let x, y, z be integers such that xRy and yRz. This means that x and y have the same digit in the 10's place, and y and z have the same digit in the 10's place. Whatever digit that is, it is the digit that is in y's 10's place, so x and z have the same digit in the 10's place. That is, xRz.

Because R is reflexive, symmetric, and transitive, it is an equivalence relation.

4) Write down one equivalence class.

$$\overline{10} = \{x \in \mathbb{Z} | x \text{ has a 1 in the 10's digit}\}$$

5) Write down the collection of all equivalence classes.

$$\{\overline{0}, \overline{10}, \overline{20}, \overline{30}, \overline{40}, \overline{50}, \overline{67}, \overline{70}, \overline{180}, \overline{90}\}$$