Define a relation $R$ on the integers via:

$$a R b \iff a \text{ and } b \text{ have the same digit in the 10's place}.$$ 

1) Give five examples of pairs of numbers that are related.

$42R40, 7R(-3), 24R123, 2202R32407, 16R112, 4R4$

2) Give five examples of pairs of numbers that are not related.

$42\not{R}50, 7\not{R}(-13), 24\not{R}143, 2202\not{R}32417, 16\not{R}162, 4\not{R}14$

3) Prove that $R$ is an equivalence relation.

**Reflexive:**
Let $x$ be an integer. Then all of the digits of $x$ are the same as the digits of itself, in particulars in the 10’s place. Hence $R$ is reflexive.

**Symmetric:**
Let $x$ and $y$ be integers such that $xRy$. That means that $x$ and $y$ have the same digit in the 10’s place, so clearly $y$ and $x$ do as well. That is, $yRx$.

**Transitive:**
Let $x, y, z$ be integers such that $xRy$ and $yRz$. This means that $x$ and $y$ have the same digit in the 10’s place, and $y$ and $z$ have the same digit in the 10’s place. Whatever digit that is, it is the digit that is in $y$’s 10’s place, so $x$ and $z$ have the same digit in the 10’s place. That is, $xRz$.

Because $R$ is reflexive, symmetric, and transitive, it is an equivalence relation.

4) Write down one equivalence class.

$$\overline{10} = \{x \in \mathbb{Z} | x \text{ has a 1 in the 10's digit}\}$$

5) Write down the collection of all equivalence classes.

$$\{\overline{0}, \overline{10}, \overline{20}, \overline{30}, \overline{40}, \overline{50}, \overline{67}, \overline{70}, \overline{180}, \overline{90}\}$$