

Codename _____ Transitions, Test 1
(Do not put your name on the test; write your name and codename on the code sheet)

An abundant number is a number whose proper factors add up to more than the number itself. For instance, 12 is abundant, but 13 is not.

Consider the statement “There is an abundant number”.

1) Write this statement into mathematical symbolism.

2) Prove this statement

3) Let P , Q , and R be statements. Prove that $[(\sim Q \Rightarrow P) \wedge (Q \Rightarrow R) \wedge (\sim R)] \Rightarrow P$

4) Prove the statement below:

$$\bigcup_{A \in \bigcap_{i \in \mathbb{Z}} \{i\}} A \times A \subseteq \{1, 6, 53\}$$

5) Prove that $\forall x, y \in \mathbb{R} (x = y = 0 \vee x^2 + y^2 > 0)$

6) A real-valued function f is said to be even iff $f(x) = f(-x)$ for all input values x . Write the statement " $f(x) = x^2 + 2$ is even" in explicit mathematical symbolism. Prove the statement.

7) Let A and B be sets. Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$

8) Prove the following statement. $\exists x \in \mathbb{R} \forall y \in \mathbb{R} \exists z \in \mathbb{Q} (xyz \neq 0 \Rightarrow x \in \mathbb{Q})$