Codename ____ Transitions, Test 2, 4/22/2014

(Do not put your name on the test; write your name and codename on the code sheet)

1) Prove that for all $n \in \mathbb{Z}_{\geq 1}$:

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

2) Define a relation R on $(\mathbb{Z} - \{0\}) \times (\mathbb{Z} - \{0\})$ via $(a, b)R(c, d)$ iff the following is true:

$$ad = bc$$

a) Give an example of two elements that are related, and another example of two elements that are not related. Ask Dr. Beyerl if they're correct. (This is so that you don't try to make a proof without first getting a feel for the objects you're working with)

b) Prove that R is an equivalence relation.

3) Let S be any set. Define a relation on $\mathcal{P}(S)$ via ARB iff $A \cap B \neq \emptyset$.

Prove or disprove each of the following:

a) R is reflexive.

b) R is symmetric

c) R is antisymmetric

d) R is transitive

e) R is total

4) Show that the function below is one-to-one.

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto 9x - 2 \end{aligned}$$

5) Show that the function below is onto.

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto 9x - 2 \end{aligned}$$

6) A binomial is a mathematical expression with two terms. In this problem we will work with the variable x and constants. A binomial is an expression of the form:

$$ax + b$$

where $a, b \in \mathbb{R}$.

a) Give 5 examples of expressions that are binomials

b) Give 5 examples of expressions that are not binomials

Now define an ordering on binomials via: $ax + b \preccurlyeq cx + d$ iff: one of the conditions are satisfied:

1) $a < c$

OR

b) $a = c$ and $b \leq d$

Show that $\preccurlyeq$ is a linear ordering.