

1) Write the following English sentence as a mathematical statement:

“For any real number  $x$ , there is an integer  $y$  that is larger than  $x$ .”

$$\forall x \in \mathbb{R} \exists y \in \mathbb{Z} (y > x)$$

2) Sketch a proof of the same statement:

“For any real number  $x$ , there is an integer  $y$  that is larger than  $x$ .”

Let  $x$  be an arbitrary real number. Choose  $y = [1 + x]$ . Then  $y \in \mathbb{Z}$  and  $y = [1 + x] \geq 1 + x > x$ . Hence For any real number  $x$ , there is an integer  $y$  that is larger than  $x$ .