(Do not put your name on the test; write your name and codename on the code sheet)

1) Show that the function f, defined below, is onto. (100 points)

$$f: \mathbb{R} \to \mathbb{R}$$
$$x \mapsto 3x + 9$$

2) Give an example of a real function that is onto but not one-to-one. (25 points)

3) Give an example of a real function that is one-to-one but not onto. (25 points)

4) In a single English sentence, give a conceptual explanation for the term "injective". (25 points)

5) Give a mathematical statement that defines the term "injective". (25 points)

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6) Prove or disprove the statement given below: (100 points)
$$\exists_{x \in \mathbb{Z}}(x \in 2\mathbb{Z} \land x > 11)$$

7) Give an example of a number that is rational. (20 points)

8) Give an example of a number that is not rational. (20 points)

9) Give an example of a number in $\mathbb{Q}$. (20 points)

10) Give an example of a number in $\mathbb{R} - \mathbb{Q}$. (20 points)

11) Give an example of a number in $\mathbb{C} - \mathbb{R}$. (20 points)

12) In a single English sentence, what does the statement below mean: (25 points)

$$\exists_{\text{person } p} \forall_{\text{kitten } k} (p \text{ is friends with } k)$$
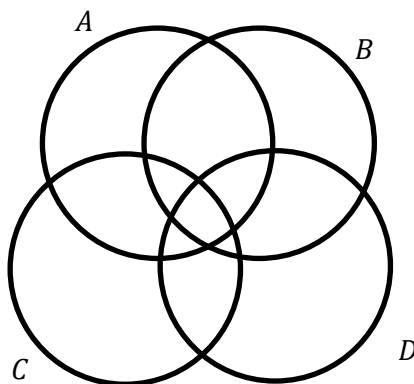
13) Find the negation of the statement below: (25 points)

$$\forall_{\varepsilon>0} \exists_{N\in\mathbb{Z}} \forall_{n>N} (|a_n - L| < \varepsilon)$$

14) Find the following union: (25 points)

$$\bigcup_{n=3}^{\infty} \left(-n, \frac{1}{n}\right)$$

15) On the Venn Diagram below, shade in the set corresponding to $(A \cap B) \cap (C - D)$. (25 points)

Let P be the statement below.

"There is a positive integer M such that for all integers n greater than M, we know that $\frac{1}{n} < 0.26$"

16) Rewrite P in formal mathematical symbols. (50 points)

17) Prove that P is true. (100 points)

18) Sketch a proof of the fact that that $\sqrt{5}$ is irrational. (100 points)