Codename ____ Transitions, Test 2

(Do not put your name on the test; write your name and codename on the code sheet)

1) Let $S = \overline{1}$ in the "mod 6" relation and $T = \overline{1}$ in the "mod 3" relation. Prove that $S \subseteq T$. (100 points)

2) Find $4 \cdot 5$ mod 13. (30 points)

3) Solve $4x \equiv 7$ mod 11. (40 points)

4) Solve $x^2 \equiv 1$ mod 4. (30 points)

5) Prove that $3|n^3 + 2n$ for all integers greater than 2. (100 points)

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6) Prove that the relation, given below, which is defined on the integers is transitive. (100 points)

$$xRy \text{ iff } x|y$$

7) Prove that the relation, given below, which is defined on the integers is not transitive. (50 points)

$$xRy \text{ iff } x|2y$$

8) Let S be an arbitrary set. Find a partition of S. (25 points)

9) Describe how we can construct an equivalence relation from a partition. (25 points)

10) A <u>weak ordering relation</u> is defined as a relation that is reflexive, antisymmetric, and transitive. Let R be the relation on the integers given by xRy iff $x \equiv_2 y$ and $x \leq y$. Sketch a proof to show that R is a weak ordering relation. (120 points)

11) Prove that for all integers $n \geq 5$ that: (100 points)

$$\prod_{m=1}^{n} \frac{1}{2m} \leq \left(\frac{1}{2^n}\right)^2$$

12) Explain what 3^{-1} means mod 7. (60 points)

13) Find 3^{-1} mod 7. (60 points)

14) Solve $3x + 2 = 6$ mod 7. (60 points)

15) How does the following LaTeX code display? (20 points)

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$x_1^2-x^2_1=y+\int_a^b t             dt$
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