Name: ____Quiz 4

Prove the equality below using induction.

$$\sum_{l=1}^{n} \frac{1}{l(l+1)} = \frac{n}{n+1}$$

Base Case: $n = 1$

$$\sum_{l=1}^{1} \frac{1}{l(l+1)} = \frac{1}{1(1+1)} = \frac{1}{2} = \frac{1}{1+1}$$

Induction Hypothesis: Assume $\sum_{l=1}^{k} \frac{1}{l(l+1)} = \frac{k}{k+1}$ for some $k \in \mathbb{N}$.

$$\sum_{l=1}^{k+1} \frac{1}{l(l+1)} = \sum_{l=1}^{k} \frac{1}{l(l+1)} + \frac{1}{k(k+1)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2)}{(k+1)(k+2)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{(k+2)} = \frac{k+1}{(k+1)+1}$$

Thus,

$$\sum_{l=1}^{k+1} \frac{1}{l(l+1)} = \frac{k+1}{(k+1)+1}$$

This proves the induction step, and thus the theorem is proven.