Name ____ Test 1, Fall 2019

Part 1: Definitions and Concepts

1) Construct a truth table for $(P \vee Q) \Rightarrow Q$
(50 points)

P	Q	$P \vee Q$	$(P \vee Q) \Rightarrow Q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

2) For the statement below, (a) Identify the individual statements and (b) Identify the statement form

"If x and y are both positive, then $x + y$ is positive."

(50 points)

P: "x is positive"
Q: "y is positive"
R: "$x + y$ is positive"

$(P \land Q) \Rightarrow R$

3) Give the definition of <u>divides</u>. Be mathematically precise and use the variables x and y to define what it means to say "$x|y$". Vague answers will be given no credit.
(50 points)

$x|y$ means that there is some $k \in \mathbb{Z}$ such that $y = kx$

4) Give the definition of even. Be mathematically precise, vague answers will be given no credit.
(50 points)

An integer n being even means that $2|n$.

Or if you want to write out the definition of divides too:

An integer n being even means that there is some $k \in \mathbb{Z}$ such that $n = 2k$.

5) Find the negation of the statement below.

(50 points)

$$\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(2x+3y=7)$$

$$\sim\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(2x+3y=7) \Leftrightarrow \exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(2x+3y\neq 7)$$

6) Determine the truth value of the statements below. (10 points each)

- $\forall_{x\in\mathbb{R}}(x \neq 0)$

False, because if $x = 0$, then $0 = 0$.

- $\exists_{x\in\mathbb{R}}(x \neq 0)$

True, because you could choose $x = 5$.

- $\forall_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy = 0)$

False, because if $x = 2$ and $y = 3$ then $6 \neq 0$.

- $\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(x^2 + xy = 0)$

True. Whatever x is, choose $y = -x$

- $\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(x^2 + xy = 0)$

True, choose $x = 0$.

Part 2: Proofs

7) Below is a partial proof of the statement below. Finish the proof.

"If $x > 2$, then $2x + 5 > 4$"

(100 points)

Line	Statement	Reasoning
(1)	$x > 2$	Premise___________________________________
(2)	$2x > 4$	Multiply line 1 by 2_________________________
(3)	$2x + 5 > 2x$	Algebra on line 2 (______________________________)

Shoot…. This is just an arithmetic fact (The left side is 5 larger than the right) and has nothing to do with line 2. This line won't be graded.

Line	Statement	Reasoning
(4)	$2x + 5 > 4$_____	Transitive property of "$<$" applied to lines 2 and 3.

8) Construct a formal proof to show that the statement below is valid.

$$\big((P \wedge Q) \wedge \big(P \Rightarrow (Q \vee R)\big) \wedge {\sim}Q\big) \Rightarrow R$$

(100 points)

Line	Statement	Reasoning
(1)	$P \wedge Q$	Premise
(2)	P	Simplification (T16) applied to line 1.
(3)	$P \Rightarrow (Q \vee R)$	Premise
(4)	$Q \vee R$	Modus Ponens applied to lines 2 and 3.
(5)	${\sim}Q$	Premise
(6)	R	Disjunctive Syllogism (T20) applied to lines 4 and 5.

9) Prove the statement below.

$$\exists_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(x + y = 7)$$

(100 points)

Choose $x = 2$ and $y = 5$. Then we have $x + y = 2 + 5 = 7$.