Name ____ Test 2, Fall 2019

Part 1: Definitions and Concepts

1) Answer true or false for each of the statements below. (6.$\bar{6}$ point each)

T or F (I) $\{3\} \in \{3\}$

T or F (II) $\{3\} \subseteq \{3\}$

T or F (III) $\{3\} \subset \{3\}$

T or F (IV) $\{3\} \in \{\{3\}\}$

T or F (V) $\{3\} \subseteq \{\{3\}\}$

T or F (VI) $\{3\} \subset \{\{3\}\}$

T or F (VII) $\{3\} \in \{3,4\}$

T or F (VIII) $\{3\} \subseteq \{3,4\}$

T or F (IX) $\{3\} \subset \{3,4\}$

T or F (X) $\{3\} = \{3\}$

T or F (XI) $\{3\} = \{\{3\}\}$

For the next couple questions, define $S = \{3\}$.

T or F (XII) $\exists_{x \in S}(x = 3)$

T or F (XIII) $\forall_{x \in S}(x = 3)$

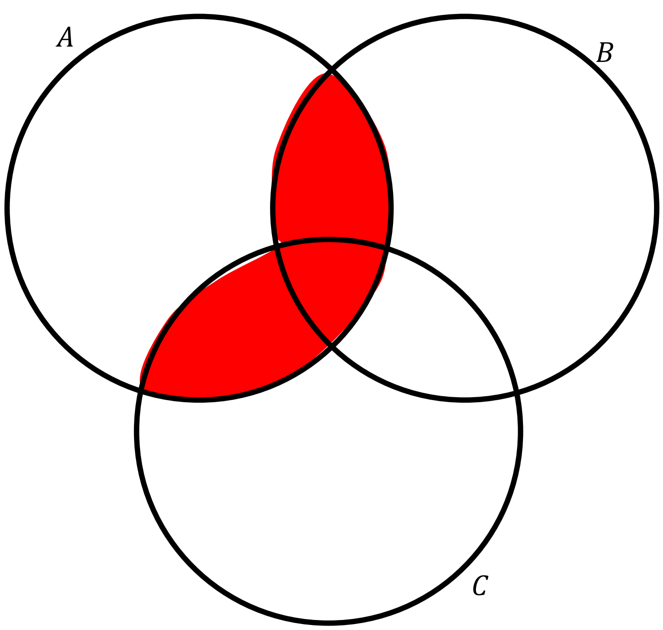
T or F (XIV) $\exists_{x \in S}(x = \{3\})$

T or F (XV) $\forall_{x \in S}(x = \{3\})$

2) Give the definition of mutually exclusive. Be mathematically precise, vague answers will be given no credit. (50 points)

Let A and B be sets. They are mutually exclusive if they have no overlap: $A \cap B = \emptyset$.

3) Shade in the region $A \cap (B \cup C)$ on the Venn Diagram below.
(50 points)



4) Find $\{2,3,4,5\} \cap \{4,5,6,7\}$
(25 points)

$\{4,5\}$

5) Find $\{2,3,4,5\} - \{4,5,6,7\}$
(25 points)

$\{2,3\}$

6) Find $[2,5] \cap [4,7]$
(25 points)

$[4,5]$

7) Find $[2,5] - [4,7]$
(25 points)

$[2,4)$

Part 2: Proofs

8) Below is a partial proof of the statement below. Finish the proof.

$$(A \cup B = B) \Rightarrow A \subseteq B$$

(100 points)

Line	Statement	Reasoning
(1)	$A \cup B = B$	Premise
(2)	Assume $x \in A$	
(3)	$x \in A \cup B$	This is because $A \subseteq A \cup B$ and line 2: $x \in A$.
(4)	$x \in B$	Combine lines 1 and 3.
(3)	$A \subseteq B$	The implication formed from lines 2-4.

9) Let A be a set. Prove that $\emptyset \times A = \emptyset$

(100 points)

Assume $\emptyset \times A$ is nonempty. Then there is some $z \in \emptyset \times A$. From the definition of cross product, we know $z = (x, y)$ for some $x \in \emptyset$ and $y \in A$. However, $x \notin \emptyset$. This is a contradiction, and so we know $\emptyset \times A = \emptyset$.

10) Assuming up to theorem T67 only, prove theorem T68. It states that:

$$(A \subseteq B \cap C \subseteq D) \Rightarrow (A \cap C \subseteq B \cap D)$$

(100 points)

Assume the premises $A \subseteq B$ and $C \subseteq D$. Suppose $x \in A \cap C$. Then because $x \in A$ and $A \subseteq B$, we know $x \in B$. Also because $x \in C$ and $C \subseteq D$, we know $x \in D$. Therefore $x \in B \cap D$. We thus have proven that $A \cap C \subseteq B \cap D$.

Note the typo. The theorem should say "$(A \subseteq B \wedge C \subseteq D) \Rightarrow (A \cap C \subseteq B \cap D)$", not what it actually says above with an intersection symbol. The proof above is for the actual theorem. Hence this question was thrown out and counted as bonus.