Name ____ Test 3, Fall 2019

Part 1: Definitions and Concepts

1) Answer true or false for each of the statements below. (10 point each)

T or F (I) A relation is always a function

T or F (II) A function is always a relation

T or F (III) Whenever we have a union of infinitely many sets, the answer is the universe.

T or F (IV) Whenever we have a union of no sets, the answer is the empty set.

T or F (V) When working mod 15, every integer can be reduced to a number in the range 0 through 14.

2) Give the definition of a relation R from a set A to a set B. Be mathematically precise, vague answers will be given no credit. (50 points)

3) Formally define the modular arithmetic relation by filing in the box below. (50 points)

$x \equiv y \bmod n$ if and only if | |

4) Given the function below, find $f(3,4)$.

$$f\colon \mathbb{R}^2 \to \mathbb{R}$$
$$(x, y) \mapsto x^2 y$$

(50 points)

5) Find all square roots of 1 in the mod 8 universe. (50 points)

6) Reduce 225 mod 13. (25 points)

7) Solve $x + 20 = 15$ mod 13 (25 points)

Part 2: Proofs

8) Below is a partial proof of the statement below. Finish the proof.

$$\forall_{j \in I}\left(A_j \subseteq \bigcup_{i \in I} A_i\right)$$

(100 points)

Line	Statement	Reasoning
(1)	Let $j \in I$ be an arbitrary index.	N/A – it's the definition of j.
(2)	Assume $x \in A_j$	N/A – It's an assumption.
(3)	_______________	Use the definition of union and the fact that $j \in I$.
(4)	$A_j \subseteq \bigcup_{i \in I} A_i$	____
(3)	$\forall_{j \in I}\left(A_j \subseteq \bigcup_{i \in I} A_i\right)$	Universal generalization on line (4) based on the fact that j was arbitrary.

9) Let R be the relation on $\mathbb{R}$ defined by xRy iff $x^2 + y^2 = 0$.

Prove that R is not reflexive.
(50 points)

Prove that R is symmetric.
(50 points)

10) Prove the inequality below for all integers $n \geq 3$, using induction.

$$n^2 - 7n + 12 \geq 0$$

(100 points)