Name ____ Test 1, Spring 2019

Part 1: Definitions and Concepts

1) Write down the truth table for $P \Rightarrow Q$
(30 points)

2) Rephrase the statement “The car will not run without gasoline in the tank” into an “if…then…” statement.
(30 points)

3) What is the definition of <u>absolute value</u>? Be mathematically precise by using an equation, something vague such as “the positive value of a number” will be given no credit.
(30 points)

4) Find the negation of the statement below.

$$\forall_{x \in A} \exists_{y \in B} (x + y > 5)$$

(30 points)

5) State the definition of an <u>even</u> number. Be mathematically precise, something vague such as "divisible by 2" will be given no credit.

(30 points)

6) State the definition of a <u>rational</u> number. Be mathematically precise, something vague such as "numbers that are fractions" will be given no credit.

(30 points)

True or False: circle the correct answer.

(10 points each)

T or F 7) The phrase "What time is dinner?" is a statement.

T or F 8) The phrase "$2 \cdot (3 + 4)$" is a statement.

T or F 9) $\big(P \wedge (P \Rightarrow Q)\big) \Rightarrow Q$ is a tautology

T or F 10) $\forall_{x \in \mathbb{R}} (2x + 3 = 7)$

T or F 11) $\exists_{x \in \mathbb{R}} \forall_{y \in \mathbb{R}} (xy = 0)$

Part 2: Proofs

12) Below is a partial proof of the statement below. Finish the proof.

$$\Big(\big((P \wedge Q) \Rightarrow R\big) \wedge (P \Rightarrow Q) \wedge P\Big) \Rightarrow R$$

(100 points)

Line	Statements	Reasoning
(1)	$P \Rightarrow Q$	Premise
(2)	P	Premise
(3)	Q	Modus Ponens applied to lines 1 and 2.
(4)	____________	____
(5)	____________	____
(6)	____________	____

13) Prove the statement below. For clarity, the statement is written twice using different symbols.

For all real numbers x, if $x > 2$, then $2x > 4$.

$$\forall_{x \in \mathbb{R}} (x > 2 \Rightarrow 2x > 4)$$

(100 points)

14) Prove the statement below.

$$\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(x < y)$$

(100 points)

15) Let a, b, and c be integers. Prove the statement below. For clarity, the statement is written twice using different symbols.

$$\text{If } a \text{ divides } b-1 \text{ and } a \text{ divides } c-1\text{, then } a \text{ divides } bc-1.$$

$$\text{If } a|b-1 \text{ and } a|c-1\text{, then } a|bc-1$$

(100 points)