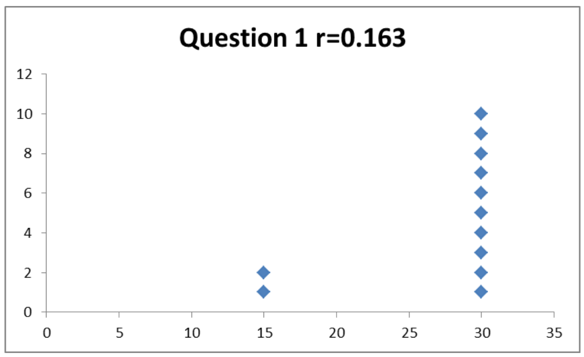
Name ____ Test 1, Spring 2019

Part 1: Definitions and Concepts

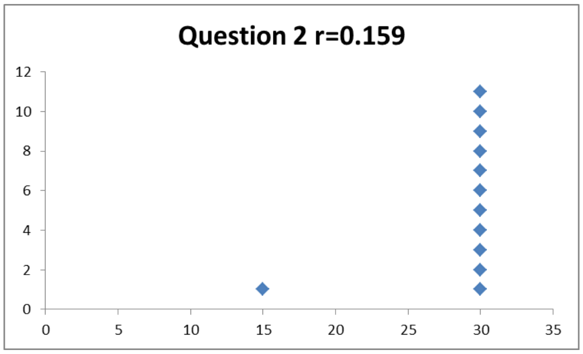
1) Write down the truth table for $P \Rightarrow Q$
(30 points)

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T



2) Rephrase the statement “The car will not run without gasoline in the tank” into an “if…then…” statement.
(30 points)

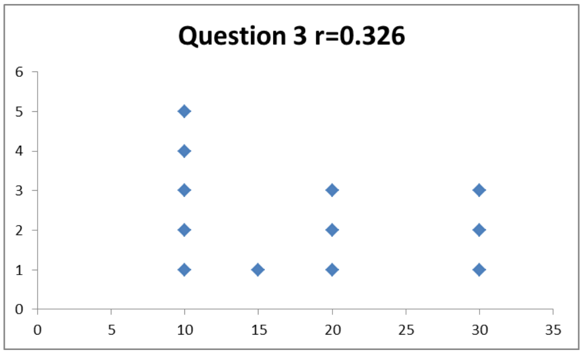
If the car does not have gasoline in the tank, then it will not run.



3) What is the definition of absolute value? Be mathematically precise by using an equation, something vague such as "the positive value of a number" will be given no credit.

(30 points)

$$|x| = \begin{cases} x, & \text{If } x \geq 0 \\ -x, & \text{If } x < 0 \end{cases}$$

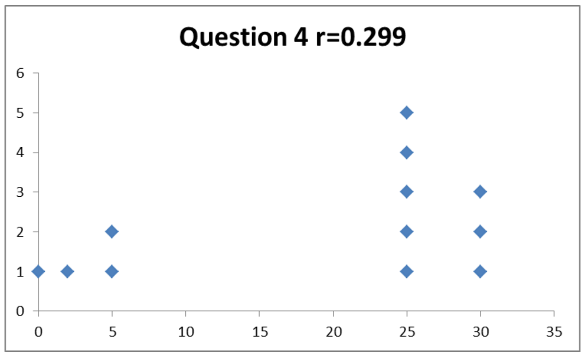


4) Find the negation of the statement below.

$$\forall_{x \in A} \exists_{y \in B} (x + y > 5)$$

(30 points)

$$\exists_{x \in A} \forall_{y \in B} (x + y \leq 5)$$

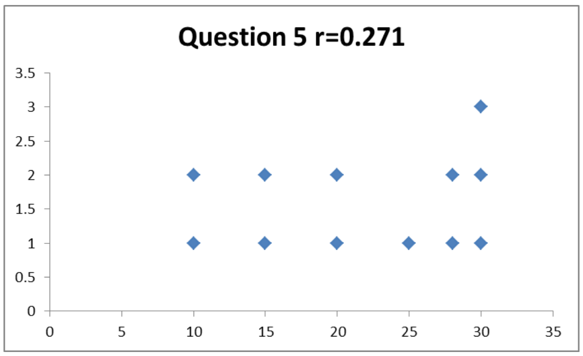


5) State the definition of an <u>even</u> number. Be mathematically precise, something vague such as "divisible by 2" will be given no credit.
(30 points)

An integer n is even if there is some $k \in \mathbb{Z}$ such that $n = 2k$.

Equivalently,

An integer n is even if $\exists_{k \in \mathbb{Z}}(n = 2k)$

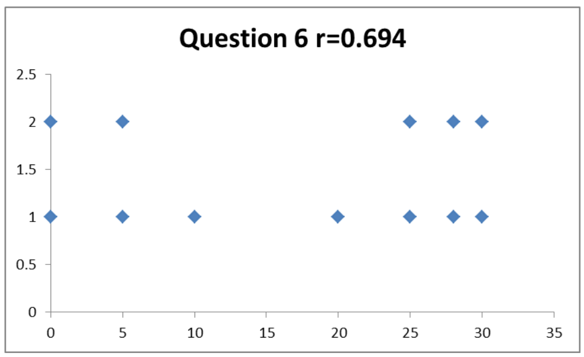


6) State the definition of a rational number. Be mathematically precise, something vague such as "numbers that are fractions" will be given no credit.
(30 points)

A number n is rational if there are some integers p and q such that $n = \frac{p}{q}$

Equivalently,

A number n is rational if $\exists_{p,q \in \mathbb{Z}} \left(n = \frac{p}{q} \right)$



True or False: circle the correct answer.

(10 points each)

T or F 7) The phrase "What time is dinner?" is a statement.

This is a question and thus does not itself have a value of true or false.

T or F 8) The phrase "$2 \cdot (3 + 4)$" is a statement.

This is a number, and thus does not itself have a value of true or false.

T or F 9) $\big(P \wedge (P \Rightarrow Q)\big) \Rightarrow Q$ is a tautology

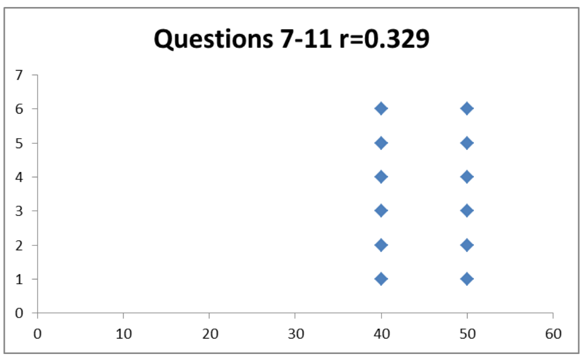
If you look at the truth table, you'll see that it's true. In fact, it's Modus Ponens and is on the theorem sheet.

T or F 10) $\forall_{x\in\mathbb{R}}(2x + 3 = 7)$

FOR ALL REAL VALUES OF x, we have that $2x + 3 = 7$. Really? No way. This is saying that $2 + 3 = 7$ and $4 + 3 = 7$ and $6 + 3 = 7$ and $14.2 + 3 = 7$ and $\frac{\pi}{2} + 3 = 7$. False. Very false.

T or F 11) $\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy = 0)$

This one is true. Choose $\boldsymbol{x = 0}$, and then for every single value of $\boldsymbol{y}$, $\boldsymbol{0 \cdot y = 0}$.



Part 2: Proofs

12) Below is a partial proof of the statement below. Finish the proof.

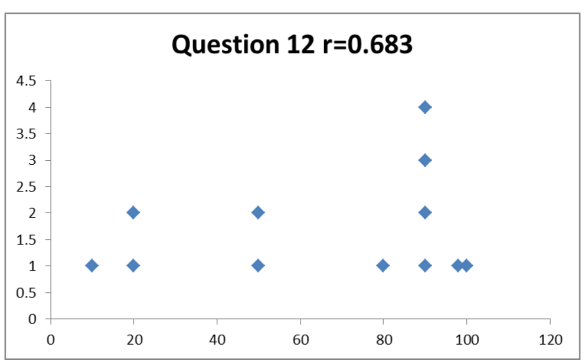
$$\Big(\big((P \wedge Q) \Rightarrow R\big) \wedge (P \Rightarrow Q) \wedge P\Big) \Rightarrow R$$

(100 points)

Line	Statements	Reasoning
(1)	$P \Rightarrow Q$	Premise
(2)	P	Premise
(3)	Q	Modus Ponens applied to lines 1 and 2.
(4)	$(P \wedge Q) \Rightarrow R$	Premise
(5)	R	Modus Ponens applied to lines 2, 3, and 4.
(6)	____________	____

Alternately you could have said:

(4)	$P \wedge Q$	Combine lines 2 and 3.
(5)	$(P \wedge Q) \Rightarrow R$	Premise
(6)	R	Modus Ponens applied to lines 4 and 5.



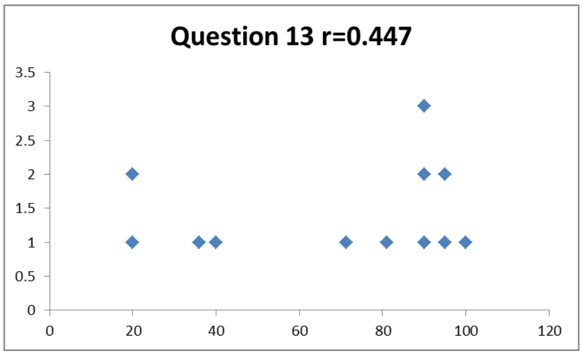
13) Prove the statement below. For clarity, the statement is written twice using different symbols.

For all real numbers x, if $x > 2$, then $2x > 4$.

$$\forall_{x \in \mathbb{R}} (x > 2 \Rightarrow 2x > 4)$$

(100 points)

See board for proof

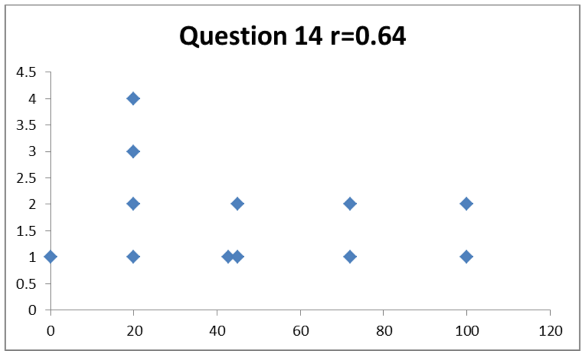


14) Prove the statement below.

$$\forall_{x \in \mathbb{R}} \exists_{y \in \mathbb{R}} (x < y)$$

(100 points)

See board for proof



15) Let a, b, and c be integers. Prove the statement below. For clarity, the statement is written twice using different symbols.

If a divides $b-1$ and a divides $c-1$, then a divides $bc-1$.

If $a|b-1$ and $a|c-1$, then $a|bc-1$

(100 points)

See board for proof