Name ____ Test 2, Spring 2019

Part 1: True or False. Circle the correct answer.

(10 points each)

T or F 1) $1 \in \{1,2\}$ True

T or F 2) $\{1\} \in \{1,2\}$ False

T or F 3) $1 \subseteq \{1,2\}$ False

T or F 4) $\{1\} \subseteq \{1,2\}$ True

T or F 5) $1 \in \{1,2\} \times \{1,2\}$ False

T or F 6) $\{1\} \in \{1,2\} \times \{1,2\}$ False

Part 2: Definitions and Concepts

For these problems let $A = \{1,2,3,4\}, B = \{3,4,5\}, C = [3,5]$ and $D = (4, \infty)$

(20 points each)

7) Find $A \cap B$.

$\{3,4\}$

8) Find $A \cup B$

$\{1,2,3,4,5\}$

9) Find $C \cap D$

$(4,5]$

10) Find $C \cup D$

$[3, \infty)$

For these problems let $A = \{1,2,3,4\}, B = \{3,4,5\}, C = [3,5]$ and $D = (4, \infty)$

(20 points each)

11) Find $D - A$

$(4, \infty)$

12) Find $B - A$

$\{5\}$

13) Draw a picture illustrating $A \times C$.

14) Draw a picture illustrating $A \times B$.

Part 3: Proofs

15) Below is a partial proof of the statement below. Finish the proof.

$$(A \cap B = A) \Rightarrow (A \subseteq B)$$

(100 points)

Line	Statements	Reasoning
(1)	$A \cap B = A$	Premise
(2)	Assume $x \in A$	
(3)	$x \in A \cap B$	Because $A = A \cap B$ and $x \in A$_______________________
(4)	$x \in B$	Because $x \in A \cap B$ means $x \in A$ and $x \in B$__________
(5)	$A \subseteq B$	The implication formed from lines 2-4.

16) Prove the statement below.

$$\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy = 0)$$

(100 points)

Choose $x = 0$. Let $y \in \mathbb{R}$ be an arbitrary real number. Therefore $xy = 0 \cdot y = 0$.

17) Prove the statement below without referencing theorem T65.

$$\text{If } A \subseteq B \text{ and } B \subseteq C\text{, then } A \subseteq C$$

(100 points)

Assume $x \in A$. Then because $A \subseteq B$, $x \in B$. Continuing, because $B \subseteq C$ we see that $x \in C$. Therefore $A \subseteq C$.

18) Prove the statement below.

$$(A - B) \cap B = \emptyset$$

(100 points)

Suppose $x \in (A - B) \cap B$. Then in particular $x \in A - B$ and $x \in B$. The first one of these tells us that $x \in A$ but $x \notin B$. We now have $x \in B$ and $x \notin B$ which is a contradiction. Therefore our assumption that such an $x \in (A - B) \cap B$ exists is false. Therefore $(A - B) \cap B = \emptyset$.