Name ____ Test 3, Spring 2019

Part 1: True or False. Circle the correct answer.

(10 points each)

Let R be the relation on $\mathbb{Z}$ given by xRy iff $(x + y)^2 = x^2 + y^2$

T or F 1) R is reflexive.

T or F 2) R is symmetric.

T or F 3) R is antisymmetric.

T or F 4) R is transitive.

T or F 5) R is an equivalence relation.

T or F 6) R is a partial order relation.

Let S be the relation on the set of real polynomials with variable x given by fSg iff $f(0) \geq g(0)$

T or F 7) S is reflexive.

T or F 8) S is symmetric.

T or F 9) S is antisymmetric.

T or F 10) S is transitive.

T or F 11) S is an equivalence relation.

T or F 12) S is a partial order relation.

Let A be a set and R a relation on A.

T or F 13) If R is an equivalence relation, it creates a partition of A.

T or F 14) If R is an partial order relation, it generalizes the idea of inequality, "$<$"

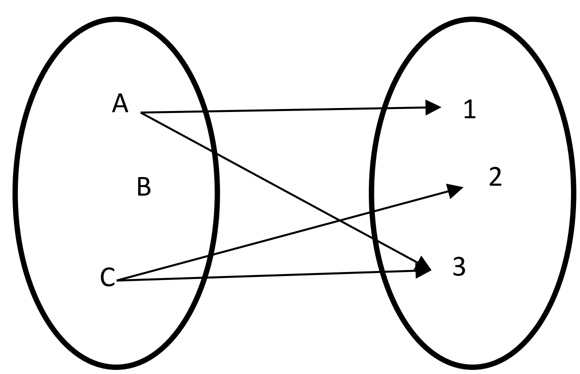
T or F 15) If P is a partition of A, then R creates the sets given by P

T or F 16) R cannot be both symmetric and antisymmetric.

Part 2: Definitions and Concepts

(20 points each)

17) For the relation diagram below, write out the relation as a set.



18) Perform the following operations in $\mathbb{Z}_{50}$.

$40 + 25 \equiv$

$15 - 40 \equiv$

19) Find $7 \cdot 8$ mod 50.

20) Solve the equation below in $\mathbb{Z}_{50}$.

$17x \equiv 1$

21) Find all “square roots of 1” mod 8.

22) Solve the equation below.

$3x + 12 \equiv 34$ mod 20

Part 3: Proofs

*****NOTE: This test may be a little long. Hence you may skip TWO of the proofs by writing "Skip". However, if you have extra time you can gamble for extra credit: for each attempted proof beyond 2, you will receive a -60 point penalty along with whatever point(s) you earn on that problem.*****

23) Let R be the relation on the set $\{A, B\} \times \mathbb{R}$ given by $(x_1, y_1)R(x_2, y_2)$ iff:

both $x_1 = x_2$ and $y_1 \leq y_2$ or both $x_1 = A$ and $x_2 = B$

It is a fact that R is a partial order relation. Below is part of that proof. Fill in the missing details.

(100 points)

Claim: R is antisymmetric.

Line	Statements	Reasoning
(1)	$(x_1, y_1)R(x_2, y_2)$	Premise
(2)	__________________	Premise

Case 1: $x_1 = x_2$:

(3)	$y_1 \leq y_2$	____
(4)	$y_2 \leq y_1$	____
(5)	$y_1 = y_2$	The antisymmetric property of $\mathbb{R}$ applied to lines 3 and 4.

Case 2: $x_1 \neq x_2$:

(6) This case is not possible because either line (1) or line (2) would be false.

(7)	$(x_1, y_1) = (x_2, y_2)$	Rephrase the conclusion from case (1), while case (2) was impossible.
(8)	R is antisymmetric	The definition of antisymmetric applied to lines (1), (2), and (7)

24) Prove the statement below for all $n \geq 1$

$$\sum_{j=1}^{n} j(j+1) = \frac{n(n+1)(n+2)}{3}$$

(100 points)

25) Prove the statement below for all $n \geq 6$.

$$n^3 < n!$$

(100 points)

26) Prove the statement below for all $n \geq 1$.

$$5 | 11^n - 6$$

(100 points)