Quiz 1 ____

Part 1: Definitions and Concepts

1) Let P and Q be true statements and R be a false statement. Answer true or false for each of the following.

T or F (I) $P \wedge Q$

T or F (II) $P \vee Q$

T or F (III) $Q \wedge R$

T or F (IV) $R \Rightarrow R$

T or F (V) $P \Rightarrow R$

2) Determine whether the following are true or false.

T or F (I) $\forall_{x\in\mathbb{R}}(x^2 + 2 \geq 0)$

T or F (II) $\exists_{x\in\mathbb{R}}(x^2 + 2 \geq 0)$

T or F (III) $\forall_{x\in\mathbb{Z}}(x + 2 = 5)$

T or F (IV) $\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(x + 1 = y)$

T or F (V) $\exists_{y\in\mathbb{R}}\forall_{x\in\mathbb{R}}(x + 1 = y)$

3) Let P be the statement "The fiddle will be played" and Q be the statement "The performer is on stage". What is the logical symbolism for "The fiddle will be played whenever the performer is on stage"?

$$Q \Rightarrow P$$

Part 2: Proofs
Some definitions and theorems are provided. Provide a proof as directed on the problem below.

(D1) $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$ for all real numbers x

(PT1) Previous Theorem 1: $|xy| = |x| \cdot |y|$ for all real numbers x and y.

(PT2) Previous Theorem 2: $|x - y| = |y - x|$ for all real numbers x and y.

4) Let a and b be real numbers such that $b \neq 0$. Prove that $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$.

Let a and b be real numbers such that $b \neq 0$. Note that we can change division to multiplication by a fraction:

$$\left|\frac{a}{b}\right| = \left|a \cdot \frac{1}{b}\right| = |a| \cdot \left|\frac{1}{b}\right|$$

Then because $1 \geq 0$, $\left|\frac{1}{b}\right| = \frac{1}{|b|}$ in all cases. Hence we conclude:

$$\left|\frac{a}{b}\right| = |a| \cdot \left|\frac{1}{b}\right| = |a| \cdot \frac{1}{|b|} = \frac{|a|}{|b|}$$