Name: ____

Part 1: Definitions and Concepts

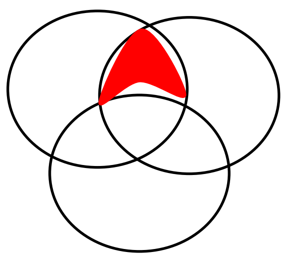
1) Let A and B be sets of real numbers. State the definition of union, in the context of A union B. Be precise: vague answers will be given no credit.

The union of A and B is the set containing everything in A, or in B, or in both.

OR

$$A \cup B = \{x \in U | x \in A \vee x \in B\}$$

2) Let A, B, and C be sets. Draw a Venn Diagram illustrating the region $(A \cap B) - C$.



3) Find the set below.

$$\bigcap_{k=5}^{\infty} \left[1 + \frac{1}{k}, 2 + \frac{1}{k}\right]$$

$$\bigcap_{k=5}^{\infty} \left[1 + \frac{1}{k}, 2 + \frac{1}{k}\right] = \left[\frac{6}{5}, \frac{11}{5}\right] \cap \left[\frac{7}{6}, \frac{13}{6}\right] \cap \left[\frac{8}{7}, \frac{15}{7}\right] \cdots = \left[\frac{6}{5}, 2\right]$$

Part 2: Proofs

4) Prove that for all natural numbers n:

$$\sum_{j=1}^{n} j \cdot 2^j = 2 + (n-1)2^{n+1}$$

Let n be a natural number.

Base Case ($n = 1$):

$$\sum_{j=1}^{1} j \cdot 2^j = 1 \cdot 2 = 2 = 2 + 0 = 2 + (1-1)2^2$$

Induction Hypothesis:

Assume $\sum_{j=1}^{k} j \cdot 2^j = 2 + (k-1)2^{k+1}$ for some $k \in \mathbb{N}$.

Induction Step:

$$\sum_{j=1}^{k+1} j \cdot 2^j = \sum_{j=1}^{k} j \cdot 2^j + (k+1)2^{k+1} = 2 + (k-1)2^{k+1} + (k+1)2^{k+1}$$

$$= 2 + k2^{k+1} - 2^{k+1} + k2^{k+1} + 2^{k+1}$$

$$= 2 + k2^{k+1} + k2^{k+1} = 2 + 2k2^{k+1} = 2 + k2^{k+2}$$

Therefore $\sum_{j=1}^{n} j \cdot 2^j = 2 + (n-1)2^{n+1}$ for all $n \in \mathbb{N}$