Name ____ Test 1, Spring 2020

Part 1: Definitions and Concepts

1) Answer each of the following as true or false. Assume P is true, Q is false, and R is true.
(10 points each)

T or F (I) $P \wedge Q$

T or F (II) $\sim P$

T or F (III) $P \vee R$

T or F (IV) $P \Rightarrow R$

T or F (V) $P \Rightarrow Q$

T or F (VI) $\forall_{x \in \mathbb{R}} (x^2 > 0)$

T or F (VII) $\exists_{x \in \mathbb{R}} (x^3 + 2x - 1 = 0)$

2) Give the definition of an <u>odd</u> integer. Be precise. Vague answers will be given no credit.
(30 points)

3) Answer each of the following as true or false. n and m are integers.
(10 points each)

T or F (I) If n is even, then $4n$ is even.

T or F (II) If n is odd, then $4n$ is odd.

T or F (III) If $n + m$ is odd, then n is odd.

T or F (IV) $\forall_{n\in\mathbb{Z}}(4n + 1\ is\ odd)$

T or F (V) $\forall_{n\in\mathbb{Z}}\exists_{m\in\mathbb{Z}}(n + m \text{ is even})$

T or F (VI) If $2|n$ and $4|m$, then $2|n + m$

T or F (VII) If $2|n$ and $4|m$, then $4|n + m$

4) Complete the definition of a <u>statement</u> below. Be precise. Vague answers will be given no credit.
(30 points)

"A statement is a complete sentence or mathematical expression that is ________________________

____"

Define the following statements for the next two problems.
P: “It is Alice’s birthday”
Q: “Alice will cry”
R: “Alice will jump for joy”

5) Write the statement “Whenever it is Alice’s birthday, she will jump for joy” in logical symbolism.
(25 points)

6) Write the statement “Alice will either cry or jump for joy” in logical symbolism.
(25 points)

Let U be the universe of people define the following open statement for the next two problems.
$P(x)$: “x has a birthday today”.

7) Write the statement “Everybody has a birthday today.” In logical symbolism.
(25 points)

8) Write the statement “Somebody has a birthday today.” In logical symbolism.
(25 points)

Part 2: Proofs

9) Below is a partial proof of the statement below. Finish the proof or create your own proof.
(100 points)

$$\big(\sim Q \land (S \lor \sim R) \land (\sim Q \Rightarrow R)\big) \Rightarrow S$$

Line	Statement	Reasoning
(1)	$\sim Q$	Premise
(2)	$\sim Q \Rightarrow R$	____
(3)	__________	Theorem 18 applied to lines 1 and 2.
(4)	$S \lor \sim R$	____
(5)	S	____

10) Prove ONE of the statements below.
(100 points)

The product of two odd numbers is odd.

OR

For all integers n: if $n|a$ and $n|b$, then $n|6a + 7b$

11) Prove ONE of the statements below.

(100 points)

$$\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy+4=4)$$

OR

For every integer, there is a larger integer.