Name ____ Test 1, Spring 2020

Part 1: Definitions and Concepts

1) Answer each of the following as true or false. Assume P is true, Q is false, and R is true.
(10 points each)

T or F (I) $P \wedge Q$

T or F (II) $\sim P$

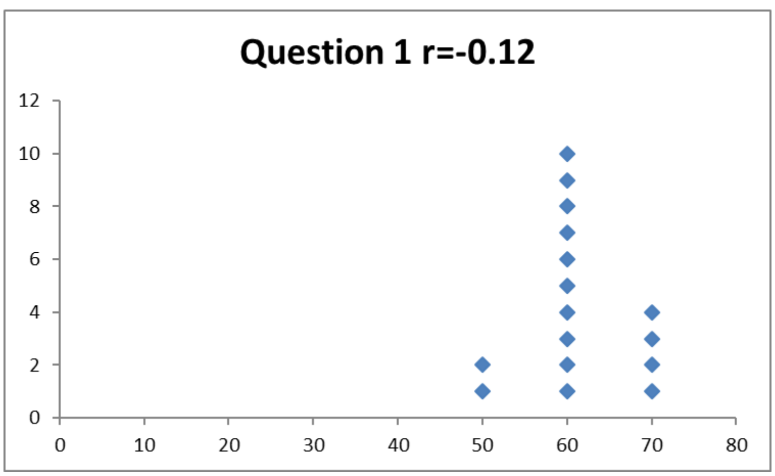
T or F (III) $P \vee R$

T or F (IV) $P \Rightarrow R$

T or F (V) $P \Rightarrow Q$

T or F (VI) $\forall_{x \in \mathbb{R}}(x^2 > 0)$

T or F (VII) $\exists_{x \in \mathbb{R}}(x^3 + 2x - 1 = 0)$



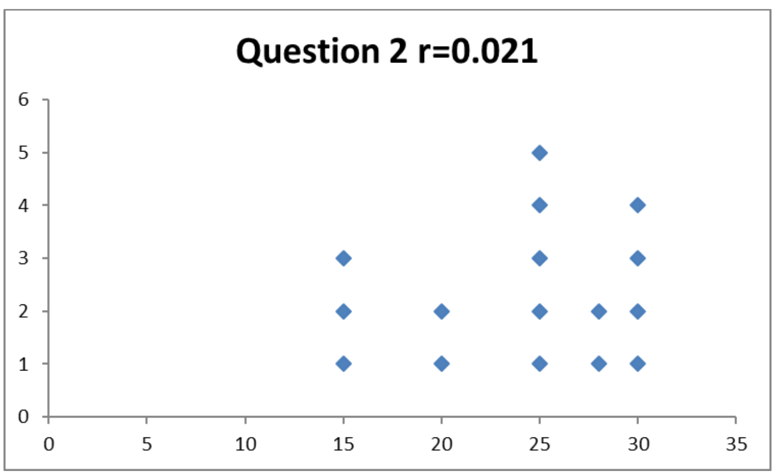
2) Give the definition of an odd integer. Be precise. Vague answers will be given no credit.
(30 points)

An integer n is odd if there is an integer k such that $n = 2k + 1$

OR

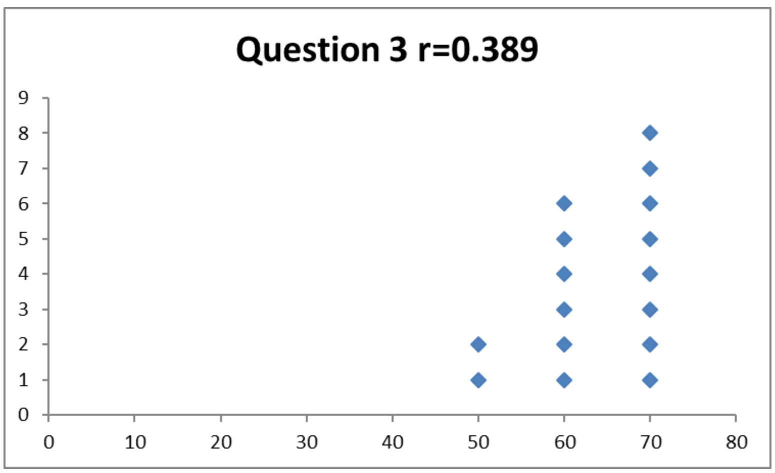
The integer n is odd if $\exists_{k \in \mathbb{Z}}(n = 2k + 1)$

***Note that you have to define n as an integer.



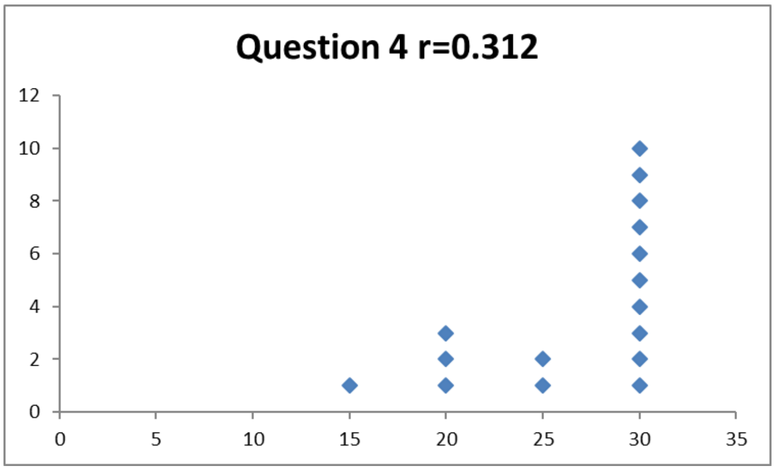
3) Answer each of the following as true or false. n and m are integers.
(10 points each)

T or F	(I)	If n is even, then $4n$ is even.
T or F	(II)	If n is odd, then $4n$ is odd.
T or F	(III)	If $n+m$ is odd, then n is odd.
T or F	(IV)	$\forall_{n\in\mathbb{Z}}(4n+1\ is\ odd)$
T or F	(V)	$\forall_{n\in\mathbb{Z}}\exists_{m\in\mathbb{Z}}(n+m \text{ is even})$
T or F	(VI)	If $2\|n$ and $4\|m$, then $2\|n+m$
T or F	(VII)	If $2\|n$ and $4\|m$, then $4\|n+m$



4) Complete the definition of a statement below. Be precise. Vague answers will be given no credit.
(30 points)

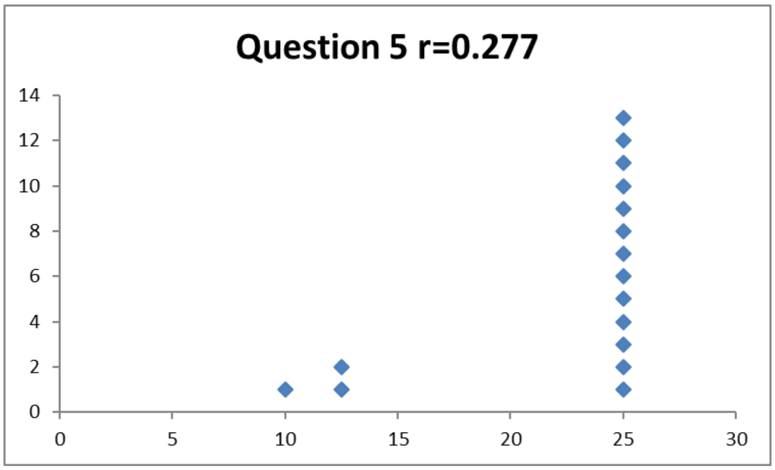
"A statement is a complete sentence or mathematical expression that is either true or false but not both."



Define the following statements for the next two problems.
P: "It is Alice's birthday"
Q: "Alice will cry"
R: "Alice will jump for joy"

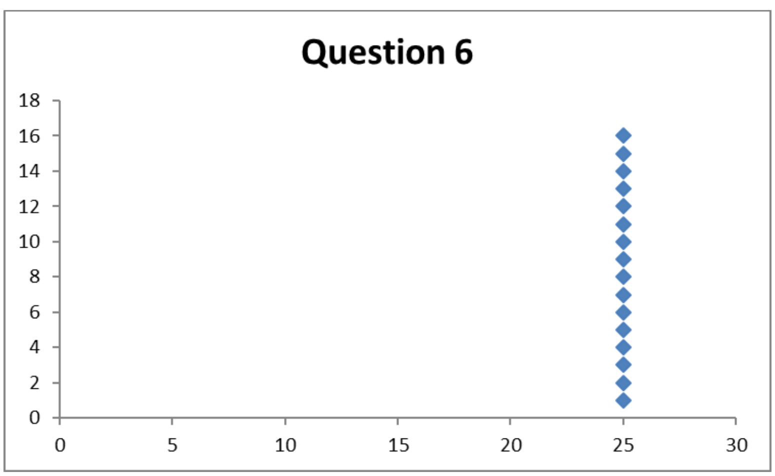
5) Write the statement "Whenever it is Alice's birthday, she will jump for joy" in logical symbolism.
(25 points)

$$P \Rightarrow R$$



6) Write the statement "Alice will either cry or jump for joy" in logical symbolism.
(25 points)

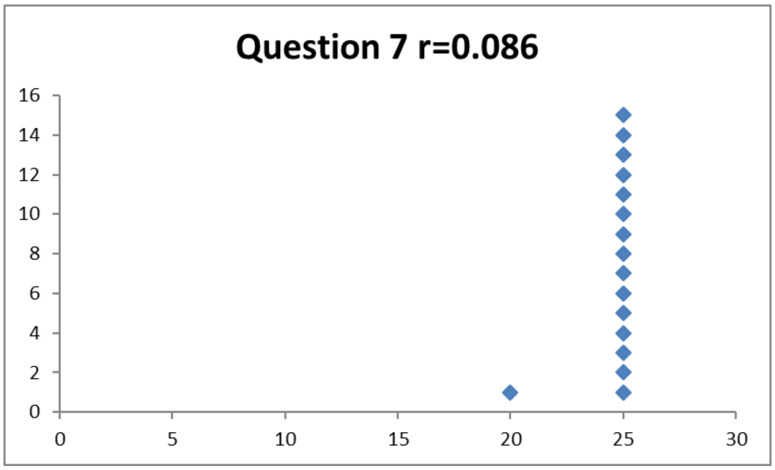
$$Q \vee R$$



Let U be the universe of people define the following open statement for the next two problems.
$P(x)$: "x has a birthday today".

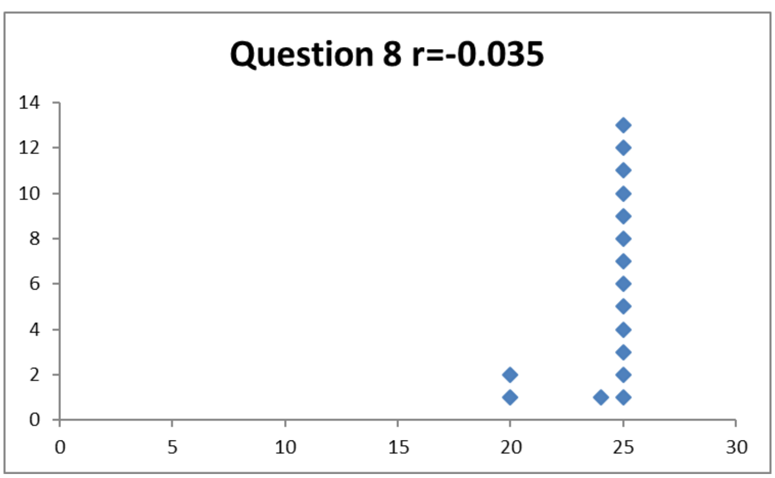
7) Write the statement "Everybody has a birthday today." In logical symbolism.
(25 points)

$$\forall_{x\in U}\big(P(x)\big)$$



8) Write the statement "Somebody has a birthday today." In logical symbolism.
(25 points)

$$\exists_{x\in U}\big(P(x)\big)$$

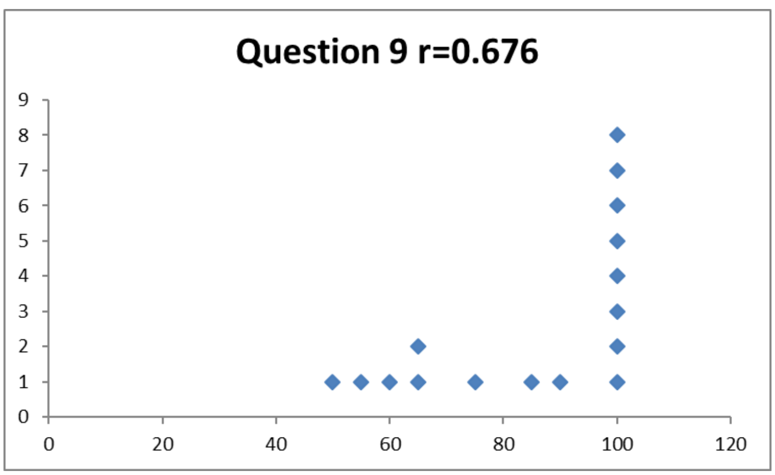


Part 2: Proofs

9) Below is a partial proof of the statement below. Finish the proof or create your own proof. (100 points)

$$\left(\sim Q \wedge (S \vee \sim R) \wedge (\sim Q \Rightarrow R)\right) \Rightarrow S$$

Line	Statement	Reasoning
(1)	$\sim Q$	Premise
(2)	$\sim Q \Rightarrow R$	Premise____
(3)	R_________	Theorem 18 applied to lines 1 and 2.
(4)	$S \vee \sim R$	Premise____
(5)	S	Theorem 20 applied to lines 3 and 4. _________________



10) Prove ONE of the statements below.
(100 points)

The product of two odd numbers is odd.

OR

For all integers n: if $n|a$ and $n|b$, then $n|6a+7b$

The product of two odd numbers is odd:
Let n and m be two odd numbers. Then we can write $n = 2k+1$ and $m = 2l+1$ for some integers k and l. Take their product to see that it is also odd:

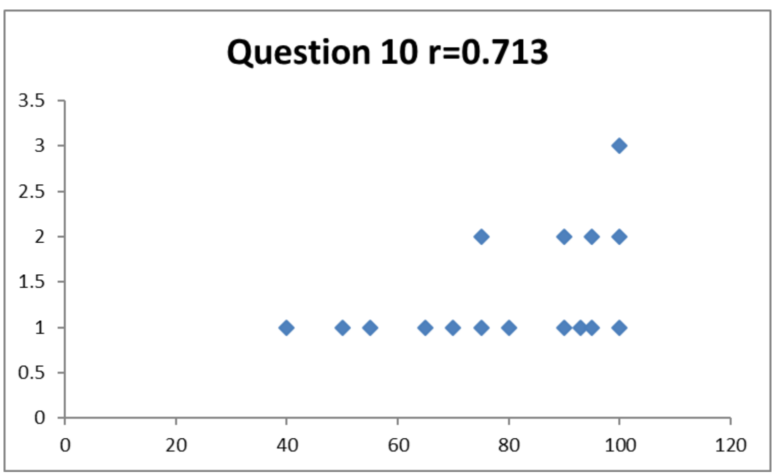
$$nm = (2k+1)(2l+1) = 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1$$

For all integers n: if $n|a$ and $n|b$, then $n|6a+7b$
Let n, a, and b be integers. Assume $n|a$ and $n|b$. Then we can write $a = nk$ and $b = nl$ for some $k, l \in \mathbb{Z}$.
Next look at the expression $6a + 7b$:

$$6a + 7b = 6nk + 7nl = n(6k + 7l)$$

Note that we see that $n|6a+7b$ from the above equation.

Key area to watch on this problem: correctly defining all your variables. Especially the ones you create from the existential. Then figuring out the right equation to look at.



11) Prove ONE of the statements below.

(100 points)

$$\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy+4=4)$$

OR

For every integer, there is a larger integer.

$\exists_{x\in\mathbb{R}}\forall_{y\in\mathbb{R}}(xy+4=4)$

Choose $x=0$ and let $y\in\mathbb{R}$. Then we get:

$$xy+4=0y+4=0+4=4$$

For every integer, there is a larger integer.

Let n be an integer and choose $m=n+1$. Note that $m>n$ because:

$$m=n+1>n$$

Key area to watch on this problem: correctly working through the quantifiers. Which variables are arbitrary (universal)? Which are chosen (existential)?