Name _____ Test 2, Spring 2020

Part 1: Definitions and Concepts

1) Illustrate $(A \cap B) \cup C$ on a Venn Diagram.
(50 points)

2) Give the definition of <u>intersection</u>, in the context of sets A and B. Be precise.
(50 points)

3) Let $A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$. Find $A - B$
(25 points)

4) Let $A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$. Find $A \cup B$
(25 points)

5) Let $A = [1,4]$ and $B = (3,6)$. Find $A - B$
(25 points)

6) Let $A = [1,4]$ and $B = (3,6)$. Find $A \cup B$
(25 points)

7) Find the intersection below.
(50 points)

$$\bigcap_{j=5}^{\infty}\left(1-\frac{1}{j}, 2-\frac{1}{j}\right)$$

8) Answer each of the following as true or false.
(10 points each)

T or F (I) Mathematical induction is used to prove a universally quantified statement.

T or F (II) Mathematical induction requires a base case.

T or F (III) The induction hypothesis in mathematical induction is an assumption.

T or F (IV) Mathematical induction proves a given statement in infinitely many cases.

T or F (V) Mathematical induction is used to prove expressions that are not statements.

Part 2: Proofs

9) Let A, B, and C be sets. Prove that $A \cap (B - C) \subseteq A \cap B$.
(100 points)

10) Prove the following statement for all natural numbers n:
(100 points)

$$\sum_{j=1}^{n} \frac{j}{(j+1)!} = 1 - \frac{1}{(n+1)!}$$

11) Using Induction, prove ONE of the statements below.
(100 points)

$n^3 < n!$ for all integers $n \geq 7$

(Hint $7^3 = 343$ and $7! = 5040$)

OR

For all natural numbers n: $8|5^{2n} - 1$