Name ____ Test 3 – Coronavirus Edition, Spring 2020

Instructions:

- The test opened at 10:00am. The test is designed for 50 minutes plus an additional 25 minutes for technical issues dealing with uploading your solutions. The test must be submitted by 11:15am.
- This test is open notes, book, internet, etc. You may use any static resources you like, but may not ask any person for assistance. As such, significantly more points will be weighted on the explanations and work than the answers themselves. Seriously, very clearly show your work, if it looks like a computer algebra system gave you an answer and you don't understand what it means, you will not receive credit.

1) Find $34 - 18 \cdot 16$ mod 12. Show your work. (10 points)

$$34 - 18 \cdot 16 \equiv 10 - 6 \cdot 4 = 10 - 24 \equiv 10 - 0 \equiv 10 \mod 12$$

***Common Mistake: Treating "mod" as an operator and not as a universe.

2) Solve $7x + 15 \equiv 42$ mod 10. Show your work. (10 points)

$$7x + 15 \equiv 42 \mod 10$$
$$7x + 5 \equiv 2$$
$$7x \equiv -3$$
$$7x \equiv 7$$
$$x \equiv 1$$

***Common Mistake: Treating "mod" as an operator and not as a universe.

3) Define the relation $\preccurlyeq$ on $\mathbb{R}$ via $x \preccurlyeq y$ if and only if $xy < 10$. Show that $\preccurlyeq$ is symmetric. (20 points)

Assume $x \preccurlyeq y$. This means that $xy < 10$. Hence $yx < 10$. Therefore $y \preccurlyeq x$. This says that $\preccurlyeq$ is symmetric.

***Common Mistake: Treating $\preccurlyeq$ as $\leq$. They are different. $\preccurlyeq$ is merely a symbol for a relation R.

4) Define the relation $\preccurlyeq$ on $\mathbb{R}$ via $x \preccurlyeq y$ if and only if $xy < 10$. Provide a counterexample to show that $\preccurlyeq$ is NOT transitive. (20 points)

We will use $x = 4, y = 2$, and $z = 3$.
$4 \preccurlyeq 2$ because $4 \cdot 2 < 10$. $2 \preccurlyeq 3$ because $2 \cdot 3 < 10$. However, $4 \not\preccurlyeq 3$ because $4 \cdot 3 \not< 10$.

***Common Mistake: Treating $\preccurlyeq$ as $\leq$. They are different. $\preccurlyeq$ is merely a symbol for a relation R.

5) Show that the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 7$ is one-to-one. (20 points)

Assume $f(x_1) = f(x_2)$
$\therefore 2x_1 + 7 = 2x_2 + 7$
$\therefore 2x_1 = 2x_2$
$\therefore x_1 = x_2$
Thus f is one-to-one.

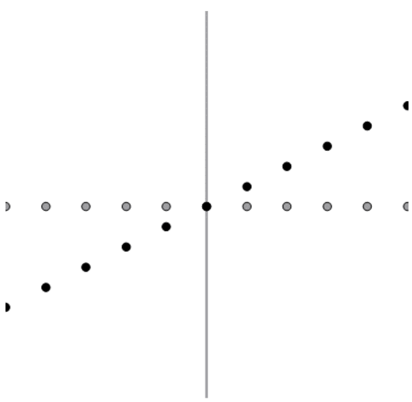
***Common Mistake: Trying to present a graph as a proof. No.

6) Find a counterexample to show that the function $f(x) = 2x^2 + 7$ is not onto. (10 points)

$f(x) \neq 0$ for any $x \in \mathbb{R}$. This is because $x^2 \geq 0$, so $2x^2 + 7 > 0$.

***Common Mistake: Trying to present a graph as a proof. No.

7) Graph the function $f: \mathbb{Z} \to \mathbb{R}$ defined by $f(x) = \frac{1}{2}x$. (10 points)



***Common Mistake: Graphing the domain as $\mathbb{R}$.