Name ____ Test 1, Fall 2021

In each part, you should skip one problem. If you make an attempt on every problem, clearly cross out the problem you do not want graded. If all are attempted and not crossed out, the last one will not be graded.

Part 1: Basic Knowledge (5 points each, 20 points total)

1) Let U be the universe, x be a variable, and $P(x)$ be an open statement. Define what the notation below means.

$$\forall_{x \in U}\big(P(x)\big)$$

2) Let A and B be sets. Define what the notation below means.

$$A \subseteq B$$

3) What is a statement? Give the definition. Be precise.

4) Let x be an integer. What does it mean for x to be even? Give the definition. Be precise.

5) Let x be a real number. What does it mean for x to be rational? Give the definition. Be precise.

Part 2: Basic Skills and Concepts (5 points each, 20 points total)

6) Simplify the statement form below by finding an equivalent statement with no need for parentheses.

$$\sim(\sim(P \vee Q) \Rightarrow R)$$

7) Fill in the missing parts of the truth table below for the statement $(P \wedge Q) \Rightarrow R$.

P	Q	R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

8) Find and simplify the negation of the statement below.

$$\forall_{x\in\mathbb{R}}\exists_{y\in\mathbb{R}}(xy = 0)$$

9) Answer each of the following as True Statement (T), False statement (F), or Not a statement (N).

T F N a) 2

T F N b) $2 + 2 = 4$

T F N c) $\forall_{x\in\mathbb{R}}(x + 2 = 4)$

T F N d) $\exists_{x\in\mathbb{R}}(x + 2 = 4)$

T F N e) $\exists!_{x\in\mathbb{R}}\,(x + 2 = 4)$

10) Given $A = \{1,2,3,4,5\}$ and $B = \{3,4,5,6,7\}$ find each of the following.

a) $A \cup B$

b) $A \cap B$

Part 3: Identifying Errors in Proofs (5 points each, 40 points total)

In each problem a formal proof is given, except that the proof is incorrect because it has one or more mistakes. Identify the first mistake by:

- Circling the line number of the mistake
- Circling the type of mistake

Other work or explanation is not required.

(They are listed in order of precedence. So if a line has multiple issues, incomprehensible should be selected before false claim, which should be selected before unjustified claim, which should be selected before incorrect logic)

11) Claim: The sum of two even numbers is even.

1.	Let x and y be even numbers	
2.	$x = 2k$ for some $k \in \mathbb{Z}$	Definition of even
3.	$y = 2k$ for some $k \in \mathbb{Z}$	Definition of even
4.	$2k + 2k$	Plug in x & y
5.	$= 2(k + k)$	Distributive property
6.	$x + y = 2(k + k)$	Transitive property of "$=$"
7.	$x + y$ is even	Definition of even

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

12) Claim: Let a and b be real numbers. $|a - b| = |b - a|$

1.	$\lvert a - b\rvert = \lvert(-1)(b - a)\rvert$	Factor
2.	$= \lvert -1\rvert \cdot \lvert b - a\rvert$	Because $\lvert ab\rvert = \lvert a\rvert \cdot \lvert b\rvert$
3.	$= \lvert b - a\rvert$	Because $\lvert -1\rvert = 1$

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

13) Claim: $2x + 3y = 6$ has a solution.

1.	Choose $x = 3$	
2.	Choose $y = 0$	
3.	$2x + 3y = 6$	Premise
4.	$2 \cdot 3 + 3 \cdot 0 = 6$	Plug in x and y
5.	$6 = 6$	Algebra

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

14) Claim: If $x > 5$, then $2x > 8$.

1. Choose $x = 6$
2. $2x = 2 \cdot 6 = 12 > 8$ — Plug in and do algebra.

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

15) Claim: Let a, b, and c be integers. If $a|b$ and $b|c$, then $a|c$.

1. $a|b$ — Premise
2. $b|c$ — Premise
3. $\frac{b}{a}$ — Division
4. $\frac{b}{c}$ — Division
5. $\frac{a}{c}$ — Plug line 4 into line 3.
6. $a|c$ — Definition of division.

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

16) Claim: If $2 \nmid x$, then $4 \nmid x$

1. Assume $4|x$ — Assumption for a contrapositive proof
2. $x = 4k$ for some $k \in \mathbb{Z}$ — Definition of division
3. $x = 2(2k)$ — Algebra
4. $2|x$ — Definition of division
5. If $2 \nmid x$, then $4 \nmid x$ — Implication from lines 1 to 4.

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

17) Claim: $\Big(\big(((Q \vee R) \Rightarrow S\big) \wedge (Q \wedge R)\Big) \Rightarrow S$

1. $Q \cap R$ — Premise
2. $(Q \cup R) \Rightarrow S$ — Premise
3. Q — Simplification (T16) on line 1
4. $Q \cup R$ — Addition (T17) on line 3
5. S — Modus Ponens (T18) on lines 2 and 4.

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

18) Claim: $x^2 + y^2 = -8$ is inconsistent.

1. Assume $a^2 + b^2 = -8$ for some $a, b \in \mathbb{R}$ — Assumption for a contradiction proof
2. $a^2 \geq 0$ — Positivity axiom
3. $b^2 \geq 0$ — Positivity axiom
4. $a^2 + b^2 \geq 0$ — Add lines 2 and 3
5. > -8 — Property of numbers
6. $-8 > -8$ — Transitivity property on lines1, 4 and 5
7. $x^2 + y^2 = -8$ — Proof by contradiction on lines 1 because $-8 \not> -8$

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

19) Claim: If n is a natural number, then $\frac{n}{n+1} > \frac{n}{n+2}$.

1. Assume n is a natural number — Premise
2. Assume $n(n+2) \leq n(n+1)$ — Assumption for the purpose of contradiction
3. $n^2 + 2n \leq n^2 + n$ — Algebra
4. $2n \leq n$ — Algebra
5. $2 \leq 1$ — Algebra
6. $\frac{n}{n+1} > \frac{n}{n+2}$ — Apply proof by contradiction because $2 \nleq 1$.

Type of Mistake: (Incomprehensible) (False claim) (Unjustified claim) (Incorrect logic)

Part 4: Proofs (10 points each, 20 points total)

20) Prove that the sum of an integer and a rational number is a rational number.

21) Let x and y be integers. Prove that if 4 does not divide $2x + y$, then either 2 does not divide x or 4 does not divide y.

22) Prove that if $x > 2$ and $2y > 1$, then $6xy > 5$