

Name _____ Test 2, Fall 2021

Part 1: Basic Knowledge (5 points each, 10 points total)

1) Let I be an index set and A_k a set for each $k \in I$. Define what the notation below means.

$$\bigcap_{j \in I} A_j$$

2) Let $f: A \rightarrow B$ be a function. Define what it means for f to be surjective, also known as onto.

Part 2: Basic Skills and Concepts (5 points each, 20 points total)

3) Answer true or false:

- a) $5 \in \{5\}$
- b) $\{5\} \in \{5\}$
- c) $5 \subseteq \{5\}$
- d) $\{5\} \subseteq \{5\}$
- e) $5 = \{5\}$

4) Multiple choice: Which of the following excerpts of LaTeX code will create the symbol below.

- a) `\sum_{k=1}^5 k^2`
- b) `$$\sum_{k=1}^5 k^2$$`
- c) `[math]\sum_{k=1}^5 k^2[math]`
- d) `\start\sum_{k=1}^5 k^2\end`

$$\sum_{k=1}^5 k^2$$

5) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 5x + 6$. Answer true or false:

- a) f is surjective
- b) f is decreasing
- c) f has an inverse
- d) f is an identity
- e) The domain of f is $5x + 6$

6) Let $A = [4,7]$ and $B = (5,12)$. Find:

- a) $A \cup B$
- b) $A \cap B$
- c) $A - B$

Part 4: Proofs (10 points each, 60 points total)

7) Let f be defined below. Prove that f is surjective.

$$\begin{aligned} f: \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto 7x - 12 \end{aligned}$$

8) Let g be defined below. Prove that g is injective.

$$\begin{aligned} g: \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto 5x + 8 \end{aligned}$$

9) Let f be defined below. Prove that f has an inverse.

$$\begin{aligned} f: \mathbb{R} &\rightarrow \mathbb{R} \\ x &\mapsto 2x \end{aligned}$$

10) Prove theorem T68 on the theorem sheet, using only earlier theorems.

11) Prove theorem T85 on the theorem sheet, using only earlier theorems.

12) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions such that $g \circ f: A \twoheadrightarrow C$. Prove that $g: B \twoheadrightarrow C$.
(Hint: Recall that \twoheadrightarrow means surjective)

Part 5: Review (5 points each, 10 points total)

13) Let P and Q be statements. Make the truth table for $P \Rightarrow Q$.

14) Prove that then $5x > 8$ whenever $x > 2$.