Name ____ Test 1, Fall 2022

Throughout the test, unless otherwise specified, you may assume P, Q, and R are statements, x, y, and z are real numbers, m, n, r and s are integers.

Part 1: Basic Knowledge

1) Give a precise definition of an <u>odd</u> integer. (5 points)

2) Give a precise definition of an <u>open statement</u>. (5 points)

3) Answer each of the following. (1 points each)

T F a) This is a universal statement: Every cat has nine lives.
T F b) This is a universal statement: Fluffly has nine lives.
T F c) This is a universal statement: Whenever fluffy falls, she expends a life.

T F d) This is an existential statement: If you score an A, you have done well.
T F e) This is an existential statement: There is somebody that has scored an A.
T F f) This is an existential statement: Michael scored an A.

T F g) $P \Rightarrow Q$ is equivalent to $Q \Rightarrow P$
T F h) $P \Rightarrow Q$ is equivalent to $\sim Q \Rightarrow \sim P$
T F i) $P \Rightarrow Q$ is equivalent to $\sim P \Rightarrow \sim Q$

T F j) $P \vee \sim P$

Part 2: Basic Skills and Concepts (5 points each, 20 points total)

4) Give the complete truth table for $P \Rightarrow \sim(Q \vee P)$.

5) Simplify $\sim\big(P \wedge (P \Rightarrow Q)\big)$ so that negations are applied only to individual statements.

6) Answer each of the following. A true/false question is true if it is *always* true and false if it is *ever* false. (1 points each)

T F a) If $x > 0$, then $|x| = x$
T F b) If $x < 0$, then $|x| = -x$
T F c) $|x| \geq 0$

T F d) If I know a statement is true, it must be true.
T F e) If a statement is true, some person must know it is true.
T F f) Mathematics proves a "God" exists because there must be must be some being that knows the truth value of all statements. We call that being "God".

T F g) $\forall_{x \in \mathbb{R}} \forall_{y \in R} (x + y = 5)$
T F h) $\forall_{x \in \mathbb{R}} \exists_{y \in R} (x + y = 5)$
T F i) $\exists_{x \in \mathbb{R}} \forall_{y \in R} (x + y = 5)$
T F j) $\exists_{x \in \mathbb{R}} \exists_{y \in R} (x + y = 5)$

Part 3: Proofs (10 points each, 60 points total)

7) Prove that for all integers n, 1 divides n. That is,

$$\forall_{n \in \mathbb{Z}} (1 | n)$$

8) Let x and y be integers. Prove that if x is odd and y is even, then $3x + 4y$ is odd.

9) Let x be a real number. Prove that if x is rational, then twice x is also rational. That is,

$$x \in \mathbb{Q} \Rightarrow 2x \in \mathbb{Q}$$

10) Let x be a real number. If $x^3 + 2x^2 < 0$, then $2x + 5 < 11$

11) Consider this supposed theorem: "If n is even, then lightsabers are real". We may have a difficult time proving this theorem, but it is hopefully pretty clear what the first two lines and last line of such a proof would likely be. Fill in those blanks below. You need not provide justification.

Proof that if n is even, then lightsabers are real

1) ____________________________

2) ____________________________

⋮

[Intermediate steps are omitted]

⋮

17) ___________________________

Now suppose we have a difficult time proving this theorem, so we turn to the *contrapositive*! We may still have a difficult time proving the theorem, but again it should be clear what the first and last two lines of such a proof would likely be. Fill in those blanks below. You need not provide justification.

Proof that if n is even, then lightsabers are real

1) ____________________________

⋮

[Intermediate steps are omitted]

⋮

13) ___________________________

14) Therefore, if n is even, then lightsabers are real

2-point Bonus: What should would 12 in the above proof likely be?

12) ____________________________