Name _____ Test 2, Fall 2022

Throughout the test, unless otherwise specified, you may assume P, Q, and R are statements, x, y, and z are real numbers, m, n, r and s are integers.

Part 1: Basic Knowledge

1) Give a precise definition of an intersection of sets A and B. (5 points)

2) Give a precise definition of the complement of a set A. (5 points)

Part 2: Basic Skills and Concepts

3) Answer each of the following. (1 points each)

T F a) Coding in LaTeX requires working with three files: Source (.tex), Goop (.goo), and Output (.pdf).

T F b) LaTeX is "What You See is What You Get", just like Microsoft Word.

T F c) LaTeX applies order of operations, just like a calculator.

T F d) Every math major will use LaTeX in their career after school.

T F e) In LaTeX, dollar signs ($) are used to create inline math, while ampersands (&) are used to create display math.

4) What types of objects can be elements of sets? (5 points)

(A) Numbers
(B) Letters
(C) Sets
(D) Both (A) and (B)
(E) All of the above

5) Let $A = [0,5]$ and $B = (2,10)$ in the universe $\mathbb{R}$. (5 points)

(A) What is $A \cup B$?

(B) What is $A \cap B$?

(C) What is $A - B$?

(D) What is A^c?

(E) How many elements does $\mathcal{P}(A)$ contain?

6) Draw an illustration of $A \cup (B \cap C)$. (5 points)

Part 3: Proofs (10 points each, 60 points total)

7) Below is a proof that $\sqrt{6} \notin \mathbb{Q}$. Well, maybe. I just copied and pasted the proof we did that $\sqrt{5} \notin \mathbb{Q}$ and replaced 5 with 6. You should be wary of anyone that claims this proof is still valid. Maybe it is, maybe it isn't. But one thing is for sure, one step(s) requires better justification than we've seen with $\sqrt{5}$ because the same reasoning doesn't work.

Which is the step(s) that *should* not be clear to a student in this class, such as you? Circle that step and explain why it's not clear.
(You don't need to justify it. We haven't covered that. I want to know why the reasoning from the other proof doesn't work.)

1) Assume $\sqrt{6} \in \mathbb{Q}$
2) $\sqrt{6} = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$
3) WLOG $\gcd(a, b) = 1$
4) $\sqrt{6} \cdot b = a$
5) $6b^2 = a^2$
6) $6|a^2$
7) $6|a$
8) $a = 6k$ for some $k \in \mathbb{Z}$
9) $6b^2 = (6k)^2$
10) $6b^2 = 36k^2$
11) $b^2 = 6k^2$
12) $6|b^2$
13) $6|b$
14) $\gcd(a, b) \geq 6$
15) $\sqrt{6} \notin \mathbb{Q}$

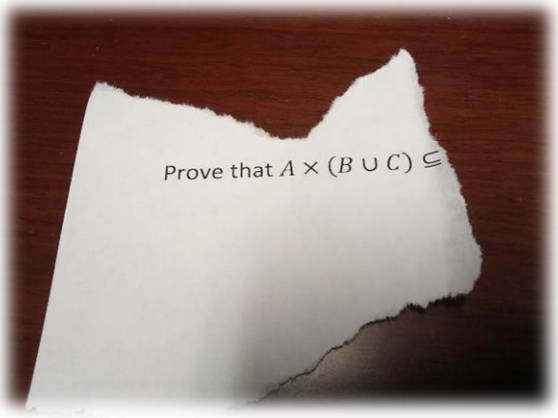
8) Prove theorem T52 on the theorem sheet, without using T52 itself or any later theorems.

9) Prove theorem T67 on the theorem sheet, without using T67 itself or any later theorems.

10) Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$

11) Prove that $x^2 = -4$ has no real solutions.

12) Alice is taking a set theory course, but unfortunately her dog ate her homework assignment! Pictured here is what is left over. Help Alice out by setting up as much of the structure and logic of this proof as you can, without knowing what the rest of the statement is.



Part 4: Review

13) Give the truth table for $P \Rightarrow Q$. (5 points)

14) Find the negation of $\forall_{x \in \mathbb{R}} \exists_{y \in \mathbb{R}} (x + y = 5)$. (5 points)