Name ____ Test 3, Fall 2022

Note that 331 is prime, in case that is relevant at some point.

Part 1: Basic Knowledge

1) What does $a \equiv b \bmod m$ mean? Hint: Don't write a sentence, just give the mathematical "tool" (5 points)

2) Let $f: A \to B$ be a function. What does it mean for f to be surjective? Give a precise definition. (5 points)
Hint: onto is a synonym of surjective.

Part 2: Basic Skills and Concepts

3) Answer each of the following. (1 points each)

T F a) $6x \equiv_{300} 1$ has a exactly 1 solution.

T F b) $6x \equiv_{300} 12$ has exactly 1 solution.

T F c) $6x \equiv_{331} 1$ has exactly 1 solution.

T F d) If $a \equiv_{331} b$, then $a \equiv_{662} b$.

T F e) If $a \equiv_{662} b$, then $a \equiv_{331} b$.

4) Find the intersection below. (5 points)

$$\bigcap_{k=1}^{\infty} \left[\frac{1}{k}, 5 + \frac{1}{k}\right)$$

5) Solve $3x + 2 \equiv 9 \bmod 10$ (5 points)

6) Solve $2x + 8 \equiv 4 \bmod 10$ (5 points)

7) Solve $330x \equiv 1 \bmod 331$ (2 bonus points)

Part 3: Proofs (10 points each, 60 points total)

8) Prove that multiplication in $\mathbb{Z}_n$ is well defined.

9) Prove the following:

$$1 \in \bigcup_{k=1}^{\infty} \left(\frac{1}{k}, 10 - \frac{1}{k} \right)$$

10) Prove the function below is injective.

$$
\begin{aligned}
f: \mathbb{R} &\to \mathbb{R} \\
x &\mapsto 8x + 2
\end{aligned}
$$

11) Prove the function below is surjective.

$$\begin{aligned} f\colon \mathbb{R} &\to \mathbb{R} \\ x &\mapsto 6x + 5 \end{aligned}$$

12) Prove the inequality below for all integers $n \geq 7$.

$$3^n < n!$$

13) Prove the equality below for all integers $n \geq 1$.

$$\sum_{m=1}^{n} (-1)^{m+1} m^2 = \frac{(-1)^{n+1} n(n+1)}{2}$$

Part 4: Review

14) Let A, B, and C be sets. Draw a Venn Diagram to illustrate $A \cap (B \cup C)$. (5 points)

15) What is the term used to describe the mistake when a proof writer assumes the conclusion they're trying to prove? (5 points)

(A) Conclusion Reasoning
(B) Concussion Reasoning
(C) Circular Reasoning
(D) Implication Reasoning
(E) Wrap-around Reasoning